ON THE QUANTIFICATION OF ROBUSTNESS WHEN COMPARING KINEMATIC INVERSION MODELS: NEW TOOLS FOR THE SOURCE INVERSION VALIDATION PROJECT

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ABSTRACT

With the goals of improving the reliability and consistency of source parameter computations, a group of researchers created the Source Inversion Validation (SIV) project. Recently a blind test exercise was held where researchers were invited to compute a slip model from ground motions generated from a synthetic model. Statistical tools are thus needed for the inter-comparison of the computed slip models as well as for the comparison of the computed slip models to the original synthetic model. In this presentation, we adapted tools used in signal processing to capture and quantify the quality of the fit when comparing different slip spatial distributions.

The tools are the 2D spatial cross spectrum and the 2D coherence function (a generalization of the coherence function used in signal processing). The two methods are complementary in their scopes and thus needed to best quantify and understand the "difference" (or misfits) between two slip models. The cross spectrum provides information that is "global" -that is, across the range of wavenumbers- as it compares and weights the relative contribution of two slip models at different wavenumbers (or wavelengths or scale lengths). On the other hand, the 2D coherence function is designed to estimate the linear dependence between the two models. The linear dependence is estimated at every wavenumber. It thus provides information that is "local in nature" -that is specific to a given wavelength- when compared to the 2D spatial cross spectrum. When applied to two slip models, the 2D coherence function allows one to measure the degree of linearity between two slips models at different wavelengths.

To take into account that the quality of the match between two slip models may depend on the areas considered, the 2D spatial cross spectrum and the 2D coherence function are computed over a moving window. We compute the 2D spatial cross spectrum and the 2D coherence function for different pairs of slip models taken from a bank of slip models computed by Shao et al. 2009 (see abstract in this volume). We also compute the 2D spatial cross spectrum and the 2D coherence function for slip models selected from the same bank and the original slip model used in the blind test exercise discussed above.

We interpret and discuss the consequences of the estimated spatial cross spectrum and of the 2D coherence in assessing the quality of fit between different slip models.

1. THE SLIP MODELS

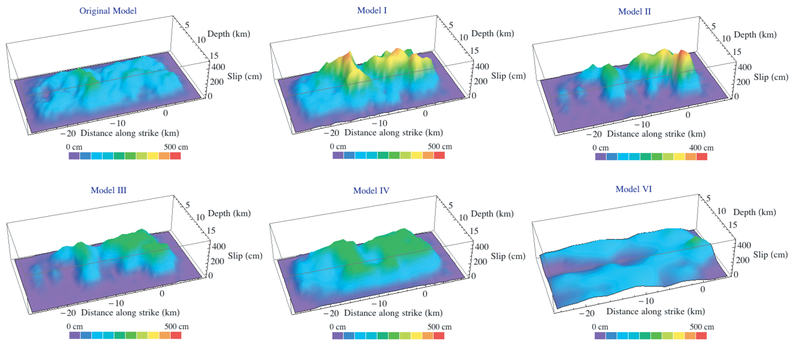

Figure 1: The spatial distribution of the original slip used in the Source Inversion Validation project (http://www.seismo.ethz.ch/staff/martin/BlindTest.html) is compared to the spatial distribution of five computed slip models. Models I to IV, computed under different hypothesis, are discussed in Shao *et al*. (2009) - see **2009 SCEC Annual Meeting poster 1-082**. The Model VI is discussed in Causse *et al*. (2008). The fault geometry in the Model VI is slightly different from the fault geometry used in Model II to IV. The scale for the vertical (slip value) axis and for the color bar is identical for all the plots.

2. THE CORRELATION COEFFICIENTS

	Original	Model I	Model II	Model III	Model IV
Original	1.000	0.922	0.684	0.723	0.818
Model I	0.922	1.00	0.724	0.761	0.841
Model II	0.684	0.724	1.000	0.925	0.641
Model III	0.723	0.761	0.925	1.000	0.711
Model IV	0.818	0.841	0.641	0.711	1.000

Table I: The correlation coefficient -also known as the product-moment coefficient of correlation or Pearson's correlation- is computed for differents pais of slip models. For this computation (and computations discuss in the Section 3), the original slip with a resolution (or sub-fault size) of 0.5 by 0.5 km is coarse grained to a resolution of 1 by 1 km. This corresponds to the resolution of the slip spatial distribution in Model I to IV and Model VI. Due to the different fault geometry used in the Model VI, Model VI is not include in this computation. According to the values reported in Table I, Models I and IV are the models that correlate best with the original slip. The highest coefficient of correlation is between the Models II and III.

3. MULTIPLE SCALE ANALYSIS: 2D SPATIAL CROSS SPECTRUM AND 2D COHERENCE FUNCTION

RATIONALES:

- **The correlation coefficient (see Section 2) is insufficient to capture the complex spatial variability of the slip distribution that involves a large range of scale lengths (Andrews, 1980)**. Analasis only based on the correlation coefficient can also be missleading. A simple example is discussed in Figure 2.
- **The frequency contains in the radiated field depends on the wavenumber distribution of the slip spatial variability (Udias, 1999)**. For instance, under the assumption that the rupture velocity is infinite, Herrero and Bernard (1994) found that the functional behavior of the Fourier amplitude of the ground motion in the angular frequency ω domain ($\propto \omega^{-2}$) mirrors the Fourier amplitude of the spatial slip heterogeneity in the wavenumber k domain (k^{-2}) where k=**k** is the 2D radial wavelength number. Limitations of this assumption is discussed in Andrews (1981) and Madariaga (1979).

When comparing two slip models, it is thus important to capture similitudes and differences that may depend on the scale length and/or the spatial position (see Figure 2).

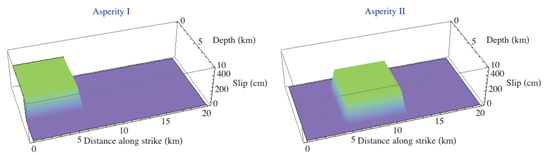

Figure 2: Two simple models of slip that only differs in the location of the asperity. The length increments are irregularly distributed. Computation of the correlation coefficient for these two asperity models gives a value close to 0. This results indicates that the correlation coefficient is not sufficient to estimate the goodness of the fit between the two aperity models.

FORMULATIONS OF THE 2D SPATIAL CROSS SPECTRUM AND 2D COHERENCE FUNCTION:

Consider two 2D fields $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$, where $\mathbf{x}$ is the posion in 2D, and their respective Fourier transform $F_1(\mathbf{k})$ and $F_2(\mathbf{k})$, where $\mathbf{k}$ is the 2D radial wavelength number, then the 2D spatial cross spectrum is given by:

$$\int d\varphi_k \, |F_1(\mathbf{k})\, F_2^*(\mathbf{k})|^2$$

where the symbol * corresponds to a complex conjugate and is φ_k is the azymutal angle in the 2D Fourier domain.

Based on the definition of the 1D coherence function (or coherency) used in time series analysis (Bendat and Piersol, 1986; Chatfield, 1989; and Nayfeh and Balachandran, 1995), the 2D coherence function at a given wavenumber **k** is given by:

$$\left(\int d\varphi_k \, |F_1(\mathbf{k})\, F_2^*(\mathbf{k})|^2\right)^2 / \left(\int d\varphi_k \, |F_1(\mathbf{k})|^2 \times \int d\varphi_k \, |F_2(\mathbf{k})|^2\right)$$

As is 1D counterpart, the 2D coherence function takes values between 0 and 1. A near zero value for the coherence function indicates that there is no linear relationship between $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ at a given wavenumber **k**. A value near one for the coherence function indicates that there is a quasi perfect linear relationship between $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ at a given wavenumber **k**. Any values between 0 and 1 will be interpreted as a partial linear coupling between $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ at a given wavenumber **k** (for after interpretations see Bendat and Piersol, 1986).

Note that the definition of coherence function used here differs from the definition adopted in Song *et al*. (2009) -see also **2009 SCEC Annual Meeting poster 2-087**.

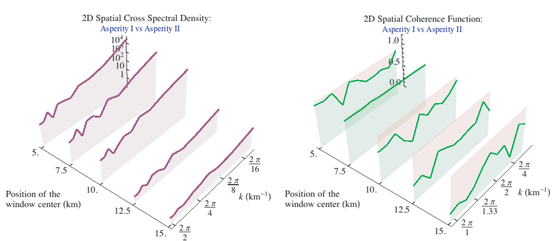

Figure 3: The 2D spatial cross spectrum and the 2D coherence function are illustrated for the asperities models illustrated in Figure 2. Both functions are computed for a moving window with its center going from 5. to 15 km by increment of 2.5 km. The moving window corresponds to a square region of 10 by 10 km^2. The behavior of the cross spectrum significantly changed when the window center is located at 12.5 or 15 km. For the window center located at 7.5 km, the coherence functions indicates a strong linear dependency between the two asperity models over all the range of wavelenghts. In the plots of the coherence function, the green area indicates the degree of linearity between the two models whereas the red area is proportional to the degree of nonlinearity between the two models.

4. 2D SPATIAL CROSS SPECTRUM AND 2D COHERENCE FUNCTION OF SLIP MODELS

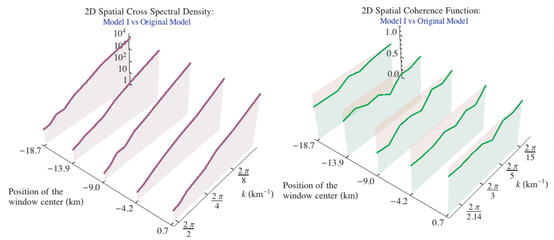

Figure 4: The 2D spatial cross spectrum and the 2D coherence function are illustrated for the original slip model and slip Model I illustrated in Figure 1. Both functions are computed for a moving window with its center located at -18.7, -13.9, -9.0, -4.2, and 0.7 km. The moving window corresponds to a square region of 15 by 15 km^2. The cross spectrum follows a power law as a function of k for all the window center. This indicates that the power law behavior embedded in the the original slip model is preserved in slip model I. For the window centered at -9.0 km, the coherence functions indicates a significant departure from linearity specially for large k values. In the plots of the coherence function, the green area indicates the degree of linearity between the two models whereas the red area is proportional to the degree of nonlinearity between the two models.

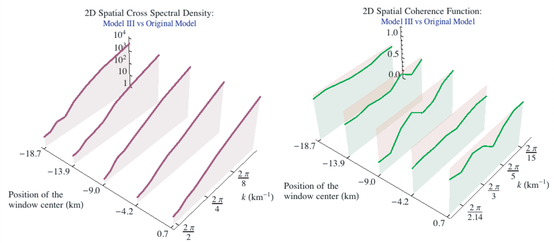

Figure 5: Same as Figure 4 but for the original slip model and slip Model III illustrated in Figure 1. For the window centered at -9.0 km, the coherence functions indicates a significant departure from linearity specially for large k values.

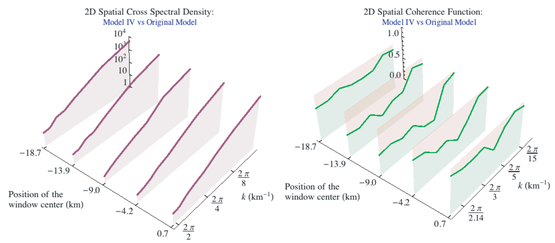

Figure 6: Same as Figure 4 but for the original slip model and slip Model IV illustrated in Figure 1. For the windows centered at-13.9 km, at -9.0 km, and at -4.2 km. the coherence functions indicates a large departure from linearity for almost all the range of k values.

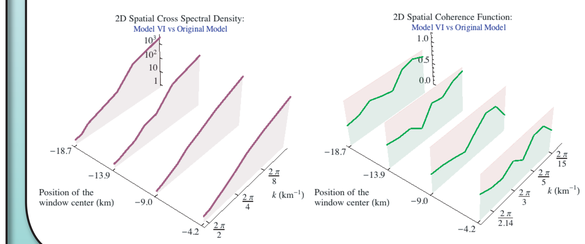

Figure 7: The 2D spatial cross spectrum and the 2D coherence function are illustrated for the original slip model and slip Model VI illustrated in Figure 1. Both functions are computed for a moving window with its center located at -18.7, -13.9, -9.0 and -4.2 km. The moving window corresponds to a square region of 15 by 15 km^2. The cross spectrum also follows a power law as a function of k for all the window center. models. For all the windows centers, the coherence functions indicates a large departure from linearity for almost all the range of k values.

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This research is supported by SCEC.

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