Source Inversion Validation Workshop at Hilton Palm Springs
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Detailed Process of Multiscale Source Inversion Analysis

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Outline

- Multiscale Source Inversion [Uchide and Ide, 2007]
  - Concept and Algorithm.
  - Synthetic test.
- My inversion process
  - How to process data and Green’s functions.
  - How to determine a source model.
  - How to determine the strength of a smoothing constraint.
- Application: The 2004 Parkfield earthquake [Uchide et al., 2009]
- Implication to the SIV project

Multiscale Source Inversion Method

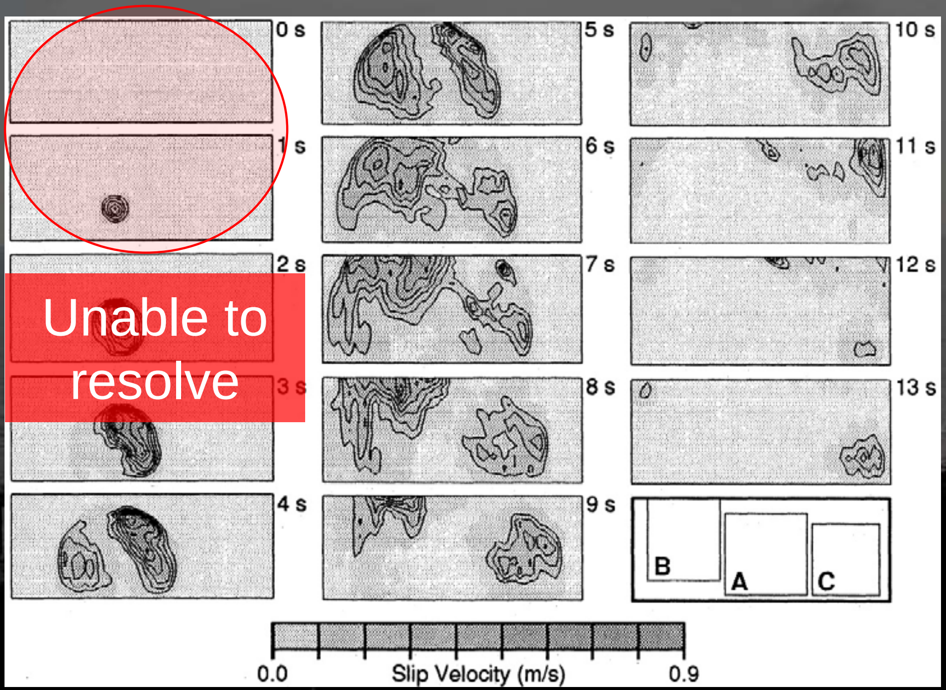
Reference:

Uchide and Ide [JGR, 2007]

Multiscale Source Inversion Method

[1] Purpose

- Early stage of large earthquake
  - Significance of initial rupture?
  - Desire a direct comparison among earthquakes of different sizes at the same scale.
- Early and main stages have been analyzed independently.
  - Analysis at a large scale.
    - Unable to resolve initial rupture.
  - Analysis only for early stage.
    - Large estimation errors (shown later).



The 1995 Hyogo-ken Nanbu earthquake [Ide and Takeo, 1997]

We developed a new slip inversion method to study throughout rupture processes stably and systematically.

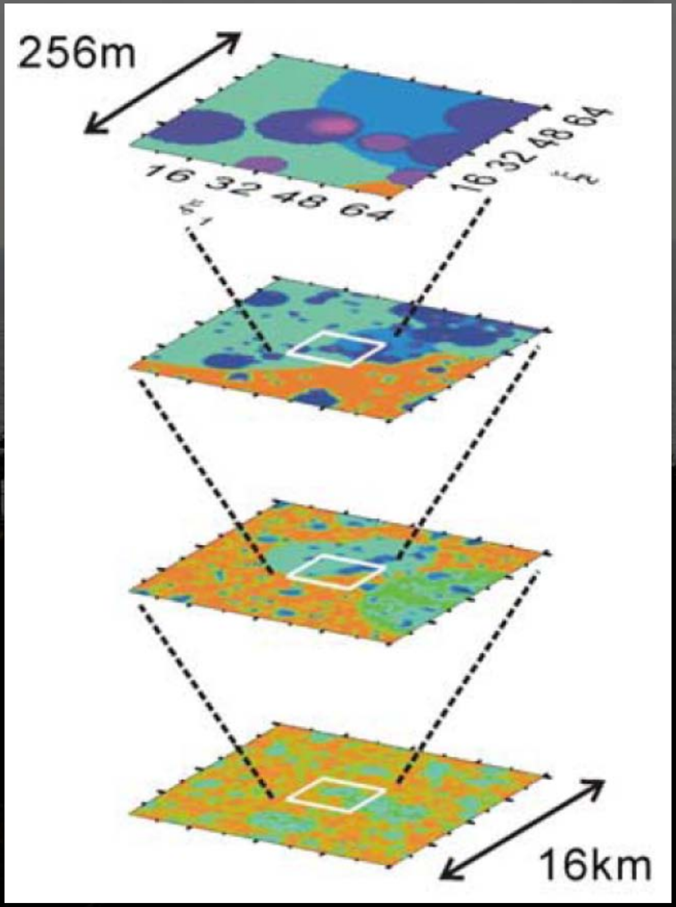
Multiscale Source Inversion Method

[2] Multiscale Source Model

- Express a source process
  in a wide scale range
  by the limited number of parameters.

- Composed of fault models with
  different grid intervals
  (namely, at different scales).

- The fault models are connected by
  renormalization.
  - Slip at large scale
    = Average slip
    in corresponding grids at small scale

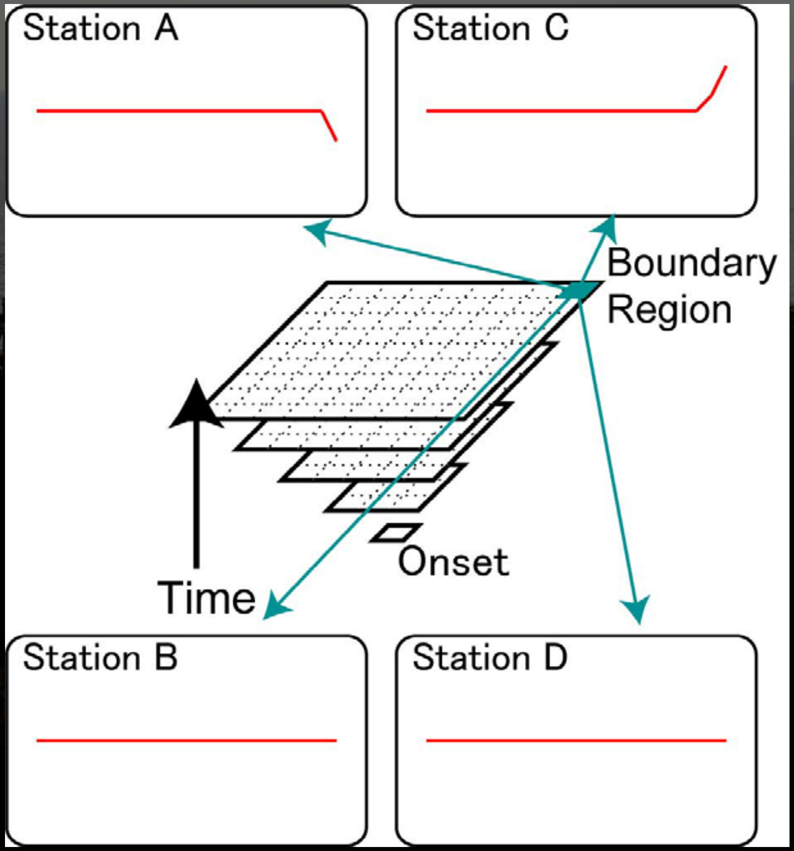


Aochi and Ide [2009]

Multiscale Source Inversion Method

[3] How to introduce the renormalization to inversion

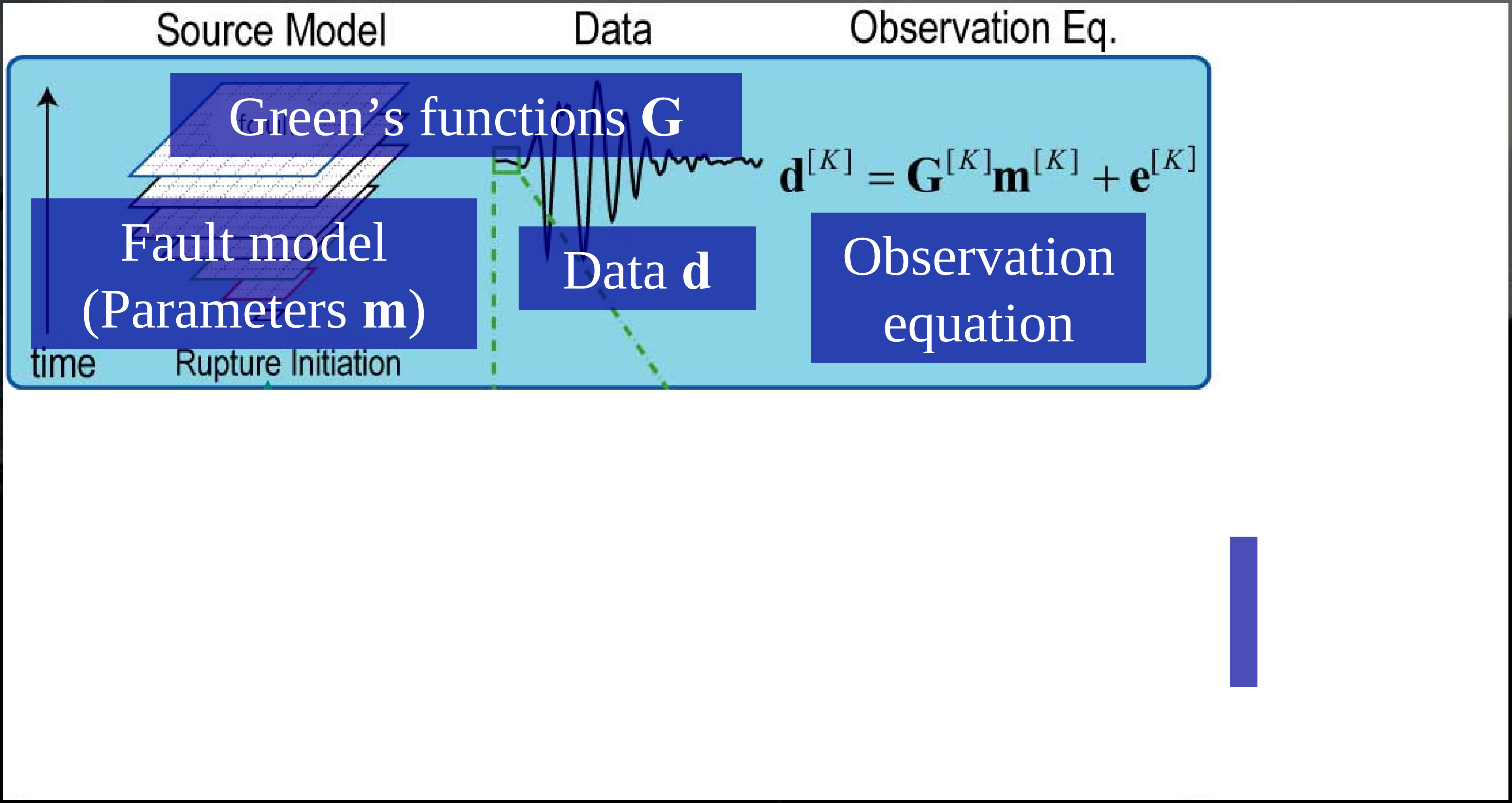
- Two choices to introduce the renormalization:
  - (1) As a constraint or a prior information, as well as a smoothing constraint.
  - (2) Introduced to the observation equation, and cannot be violated.



- We chose (2).
  - At small scale, the boundary region of the model is constrained poorly by data.
  - Choice (2) stabilizes the analysis.

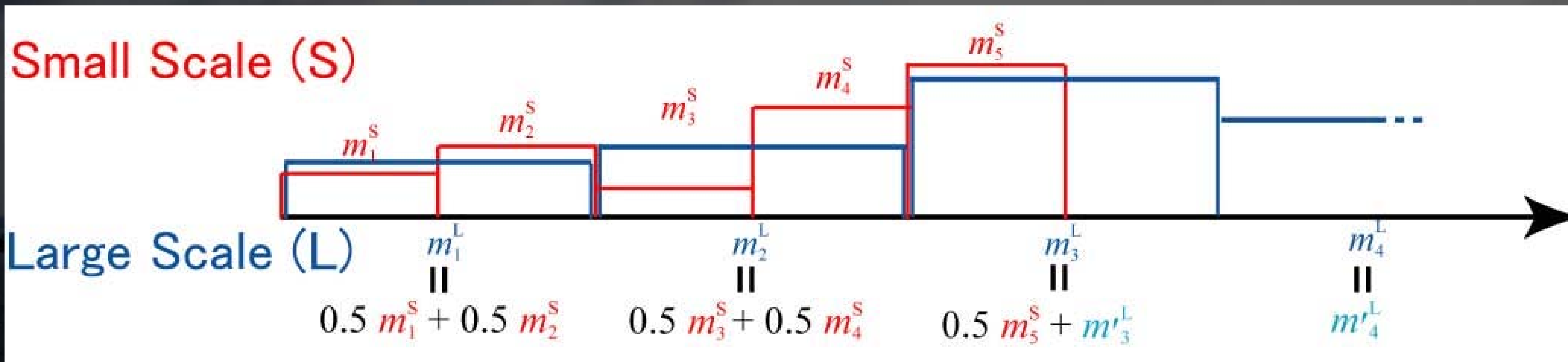
Multiscale Source Inversion Method

[4] How to combine the models at different scales



Multiscale Source Inversion Method

[5] How to introduce the renormalization to inversion



Linear Operation

We reduce the components of $\mathbf{m}^L$ totally explained by the components of $\mathbf{m}^S$. (ex., m_1^L and m_2^L above)

$$\mathbf{m}^L = \mathbf{A}\mathbf{m}^S + \mathbf{C}\mathbf{m}'^L$$

$$\mathbf{m}^S = \mathbf{m}^S$$

$$\begin{pmatrix} \mathbf{m}^S \\ \mathbf{m}^L \end{pmatrix} = \begin{pmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{A} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{m}^S \\ \mathbf{m}'^L \end{pmatrix}$$

$$\mathbf{d}^S = \mathbf{G}^S \mathbf{m}^S$$

$$\mathbf{d}^L = \mathbf{G}^L \mathbf{m}^L$$

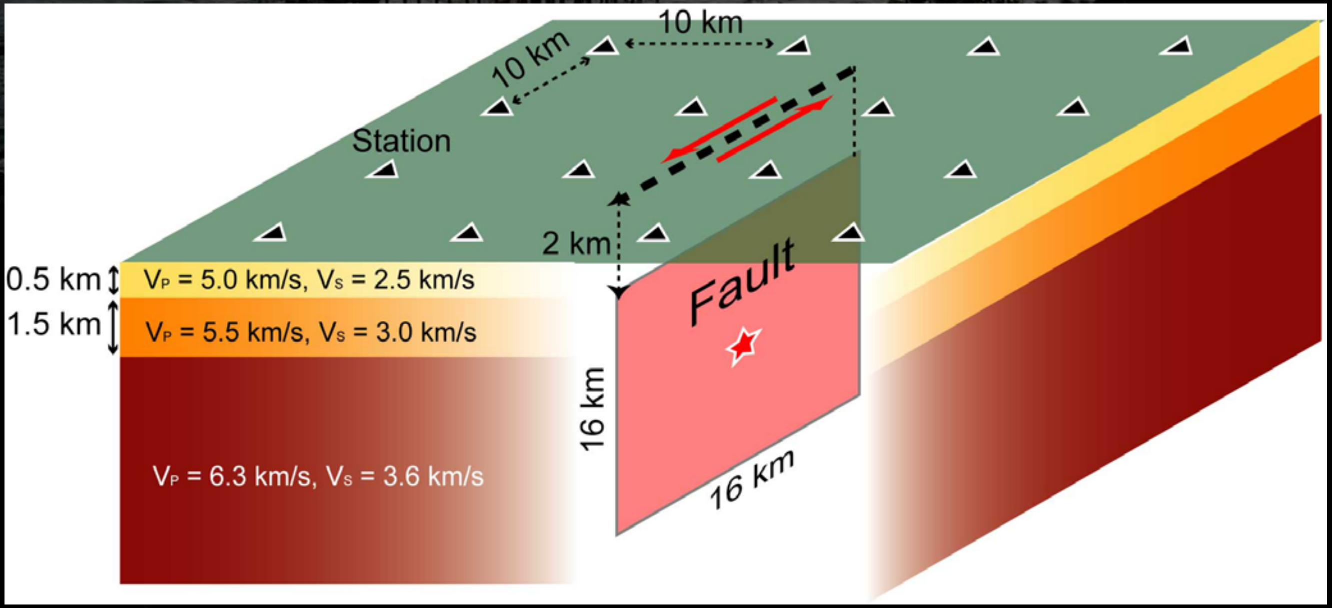
$$\begin{pmatrix} \mathbf{d}^S \\ \mathbf{d}^L \end{pmatrix} = \begin{pmatrix} \mathbf{G}^S & \mathbf{O} \\ \mathbf{O} & \mathbf{G}^L \end{pmatrix} \begin{pmatrix} \mathbf{m}^S \\ \mathbf{m}^L \end{pmatrix} = \begin{pmatrix} \mathbf{G}^S & \mathbf{O} \\ \mathbf{G}^S\mathbf{A} & \mathbf{G}^L\mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{m}^S \\ \mathbf{m}'^L \end{pmatrix}$$

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

Multiscale Source Inversion Method

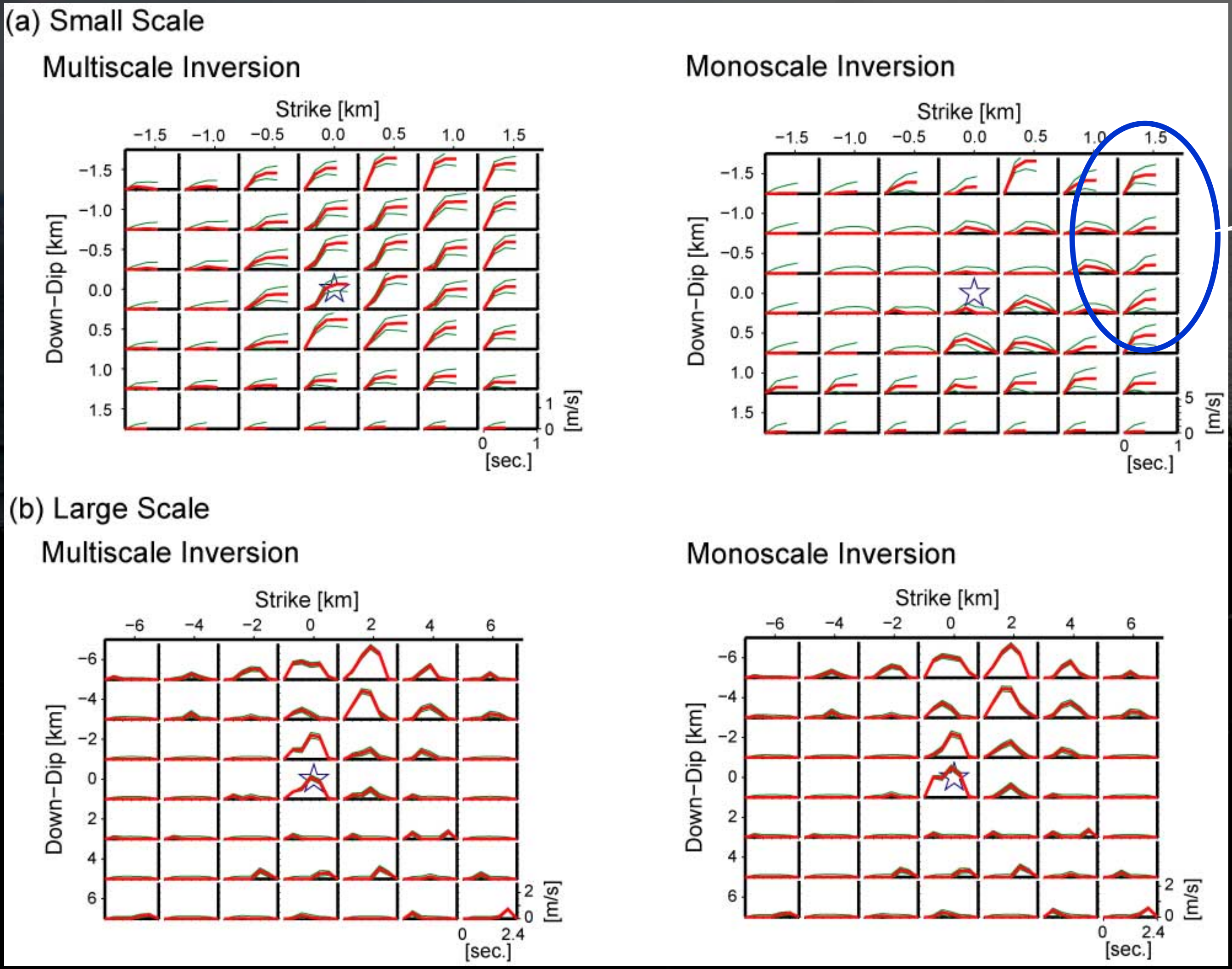
[6] Synthetic test (1/2)

- Fault geometry is given.
- 16 stations
- Underground structure
  - Forward: 3-layer model
  - Inversion: 2-layer model



Multiscale Source Inversion Method

[7] Synthetic test (2/2)



Too high slip rate & Large errors.

Red: Estimated local slip rate.

Green: Local slip rate ± Estimation error

My Inversion Process

References on ABIC:

1. Yabuki and Matsu’ura [GJI, 1992]
2. Yoshida et al. [PEPI, 1989]

My Inversion Process

[1] Data

- Data
  - Select stations
  - Determine the weight (option)
    - Compensate the azimuth coverage.
  - Pick P and/or S arrivals.
  - Resampling
    - Anti-alias filter.
    - Resampling rate:
      4 – 5 times higher than high-frequency limit of the bandpass filter.
  - Applying a bandpass filter.
  - Integral (option).

- Passband of the filter
  - Theoretical Green’s functions
    - As far as the Green’s function explain the seismograms of nearby smaller events.
    - Usually, lower than 1 Hz.
  - EGF
    - Lower than the corner frequency of EGF events.

My Inversion Process

[2] Green's functions

- Green's functions
  - Theoretical
    - Calculation by Takeo [1985]
    - Each component is time-shifted to adjust the P or S arrival
  - Empirical
    - Selecting appropriate events.
      - Located close to the target event.
      - Mechanism is similar to that of the target event.
    - Pick P and/or S arrival carefully.
    - Determine the relative origin time and location to those of the target event.
      (if time adjusting is by the origin time)

My Inversion Process

[3] Source Model

- Strike, Dip, and Rake
  - Based on aftershock distributions, CMT solutions, etc.
- Rupture initiation point
  - Hypocenter in a catalog (relocation catalog is preferable)
- Length and Width
  - Initially, source dimension expected by a scaling law.
  - Shrink or expand by trial and error to comparable to the slip area.
- Node interval of slip basis functions
  - Comparable to the minimum wavelengths of the applied bandpass filters.
- Hypothetical rupture velocity
  - Propagation speed of the hypothetical rupture front, within which slip is allowed.
  - Determined to cover the timing and position of significant slip.

My Inversion Process

[4] Algorithm

- Non-Negative Least Square (NNLS) [Lawson and Hanson, 1995]
  - Exclude parameters to be negative.
  - Practical method.
  - Statistical meaning is unclear.

- Temporal smoothing constraint
  - As prior information of Bayesian modeling.
  - Define a hyper parameter as the strength of the smoothing.
  - Preferable value of hyper parameter is to minimize ABIC.

My Inversion Process

[5] AIC (Akaike's Information Criterion)

- Entropy maximization principle
  → To maximize log likelihood.
  - Likelihood: $L(\mathbf{m}; \mathbf{d})$
    = Posterior PDF: $p(\mathbf{d}; \mathbf{m})$

- The bias of different models is approximated by # of parameter.

- AIC = $-2 \max \ln L(\mathbf{m}; \mathbf{d}) + 2M$
  (M: the number of model parameters)

Dr. Hirotugu Akaike
(1927 – 2009)

(from his website: http://tswww.ism.ac.jp/kitagawa/HTML-new/Akaike/profile.html)

My Inversion Process

[6] ABIC (Akaike's Bayesian Information Criterion)

- Smoothing constraints as the prior information of Bayesian modeling.
  - Hyper parameters α : strength of smoothing
  - Prior probability density function: $\pi(\mathbf{m}; \alpha)$
  - Likelihood: $L(\mathbf{m}, \alpha; \mathbf{d}) = p(\mathbf{d}; \mathbf{m})\, \pi(\mathbf{m}; \alpha)$

$$\mathrm{ABIC} = -2 \max \ln \tilde{L}(\boldsymbol{\alpha}; \mathbf{d}) + 2C$$

$$\left(\begin{array}{l} \text{marginal likelihood} : \tilde{L}(\boldsymbol{\alpha}; \mathbf{d}) = \int L(\mathbf{m}, \boldsymbol{\alpha}; \mathbf{d})\, d\mathbf{m} = \int p(\mathbf{d}; \mathbf{m})\, \pi(\mathbf{m}; \boldsymbol{\alpha})\, d\mathbf{m} \\ C : \text{the number of constraints} \end{array} \right)$$

- Application to source inversion
  - Geodetic: Yabuki and Matsu'ura [GJI, 1992]
  - Seismic: Yoshida et al. [PEPI, 1989]

My Inversion Process

[7] How to Calculate ABIC

$$\text{Observation equation}: \begin{pmatrix} \mathbf{G} \\ \alpha\mathbf{H} \end{pmatrix} \mathbf{m} = \begin{pmatrix} \mathbf{d} \\ \mathbf{d}_c \end{pmatrix} + \begin{pmatrix} \mathbf{e} \\ \mathbf{e}_c \end{pmatrix}$$

$$\left(\text{Prior Information}: \mathbf{Hm} = \mathbf{d}_c + \mathbf{e}_c\right)$$

↓

$$\text{ABIC} = (N + N_c - M)\ln s(\hat{\mathbf{m}}) - N_c \ln \alpha^2 + \ln\left|\mathbf{G}^t\mathbf{G} + \alpha^2\mathbf{H}^t\mathbf{H}\right| + 2C$$

$$\begin{pmatrix} s(\hat{\mathbf{m}}): \text{residual} \\ N: \#\text{ of data}; N_c: \#\text{ of constraint data}; M: \#\text{ of parameters} \end{pmatrix}$$

Yabuki and Matsu'ura [GJI, 1992]

Application: The 2004 Parkfield earthquake

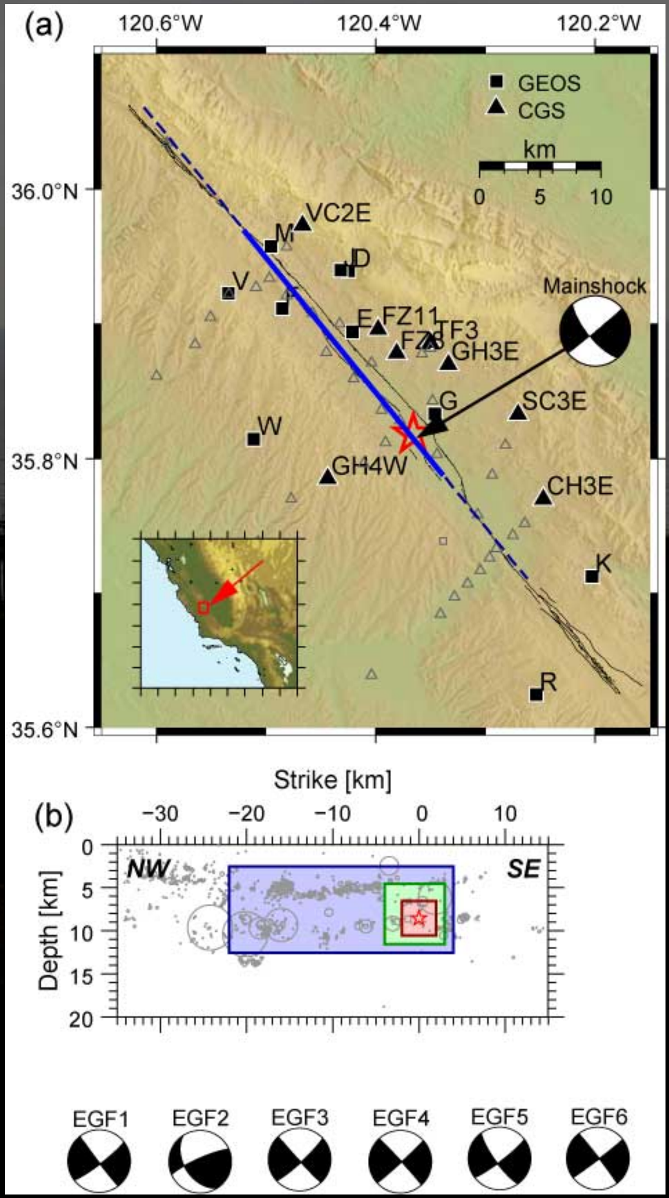
Reference:

Uchide et al. [GRL, 2009]

The 2004 Parkfield Earthquake

[1] Conditions of the analysis

- Multiscale fault model
- Data
  - Velocity seismogram
    - By integrating acceleration records.
  - Band-pass filter
    - Small Scale: 2.0 – 10.0 Hz
    - Medium Scale: 1.0 – 5.0 Hz
    - Large Scale: 0.05 – 0.25 Hz
- Green's functions
  - Small and Medium Scales: EGF
  - Large Scale: Theoretical Green's functions.
- Constraints
  - Temporal smoothing
  - Non-negative slip rate
    - NNLS [*Lawson and Hanson*, 1995]
  - Total M_o equivalent to M_w 6.0

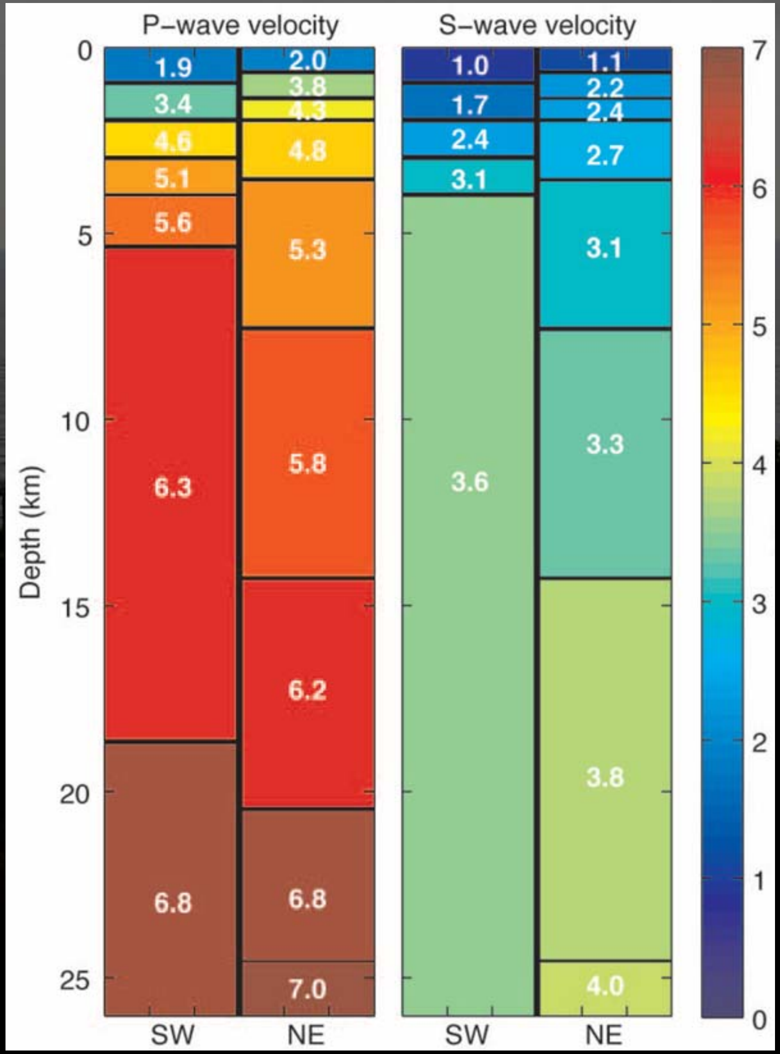


The 2004 Parkfield Earthquake

[2] Theoretical Green's Functions for the Large Scale

- Assuming 1-D layered structure
  - NE and SW stations [*Liu et al.*, 2006] based on the result of the DD tomography [*Thurber et al.*, 2006]

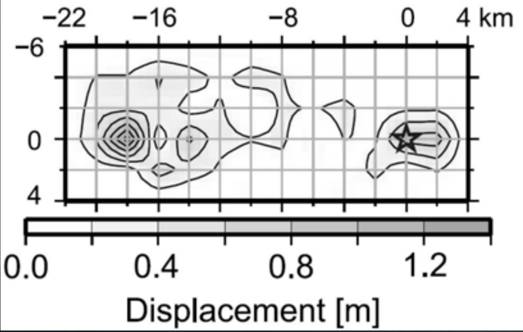
- Algorithm
  - Reflection-Transmission matrices [*Kennett and Kerry*, 1979]
  - Discrete wavenumber integral [*Bouchon*, 1981]
  - Anelastic effect by the use of complex velocities [*Takeo*, 1985]



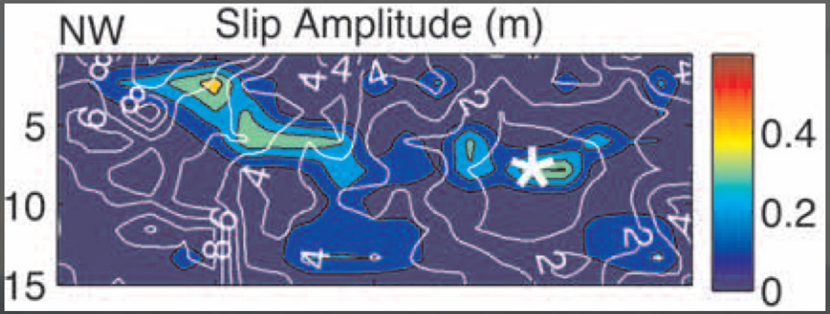
Liu et al. [2006]

The 2004 Parkfield Earthquake

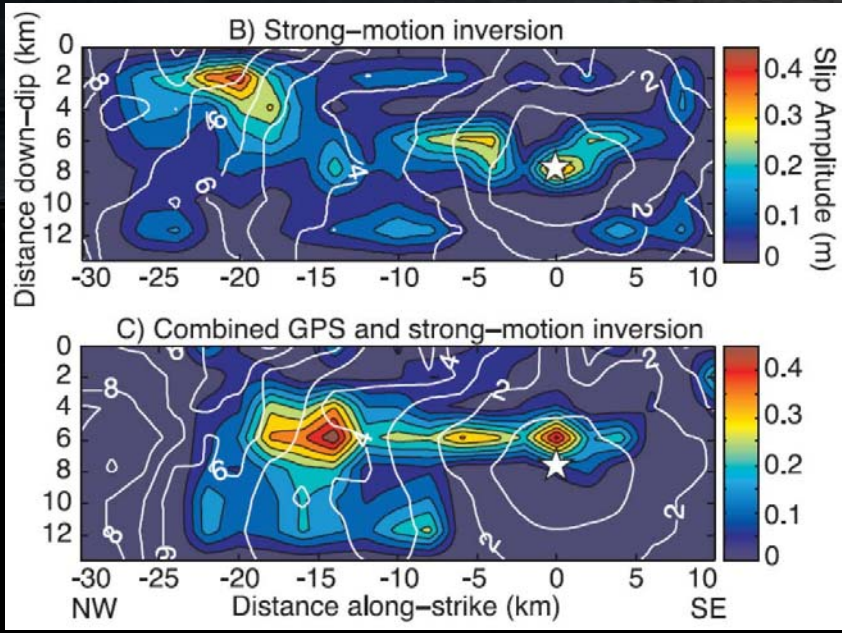
[3] Final Slip



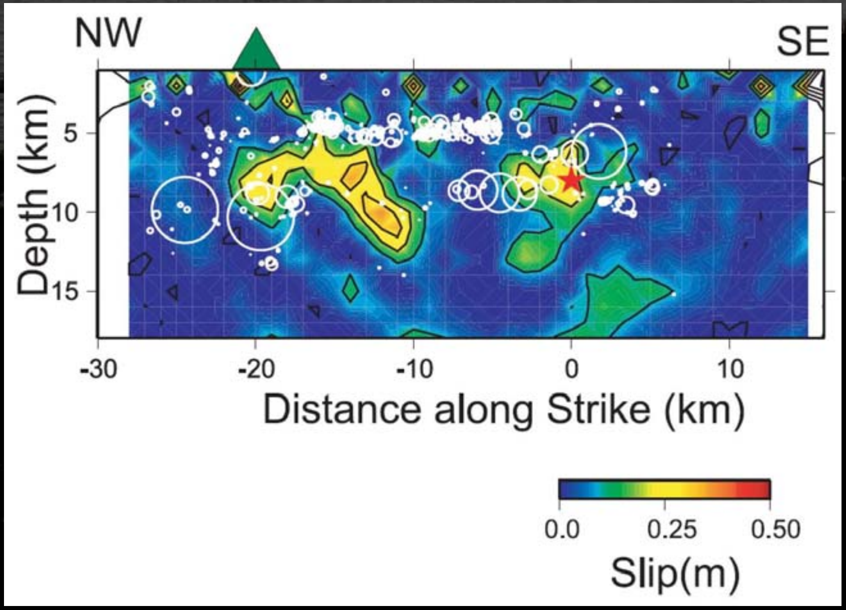
Uchide et al. [2009]



Liu et al. [2007]



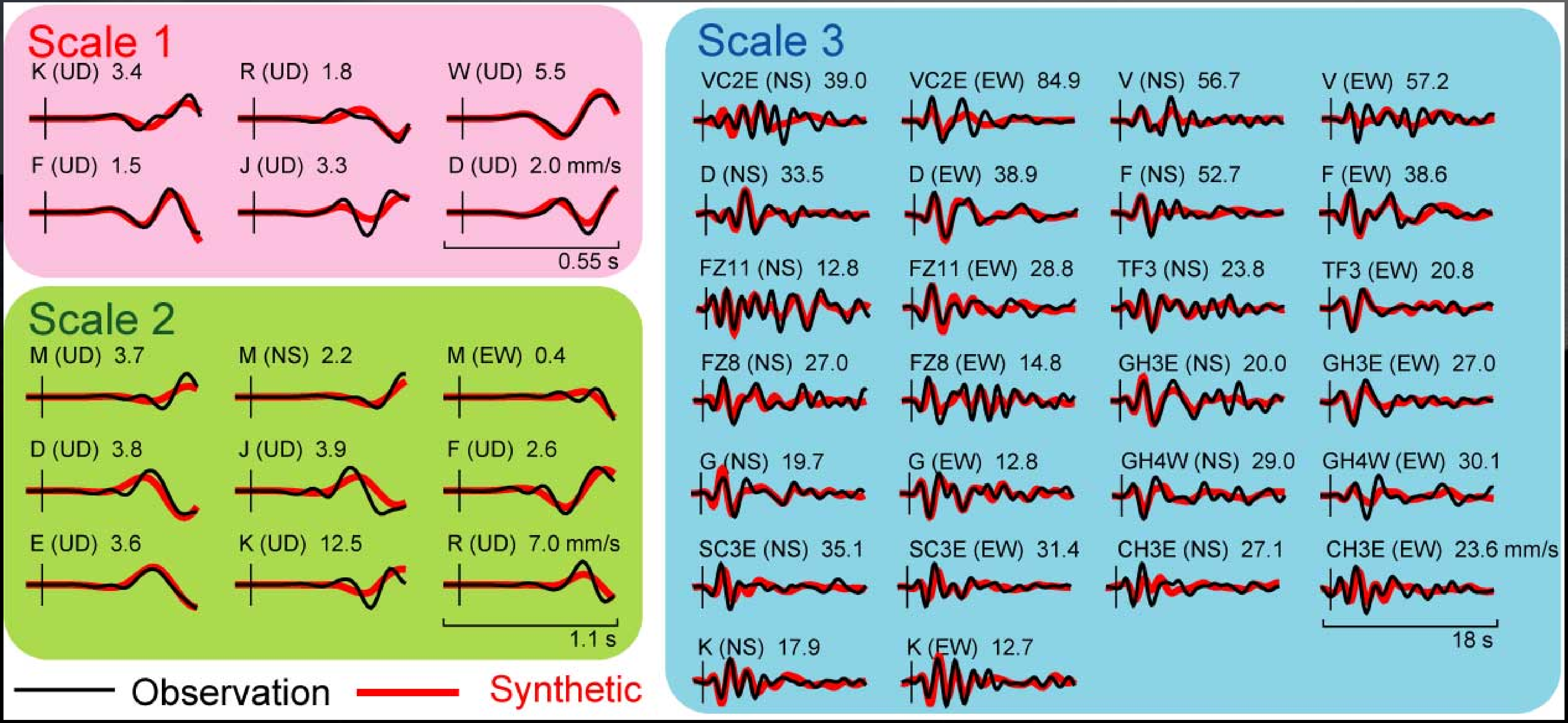
Custódio et al. [2009]



Kim and Dreger [2008]

The 2004 Parkfield Earthquake

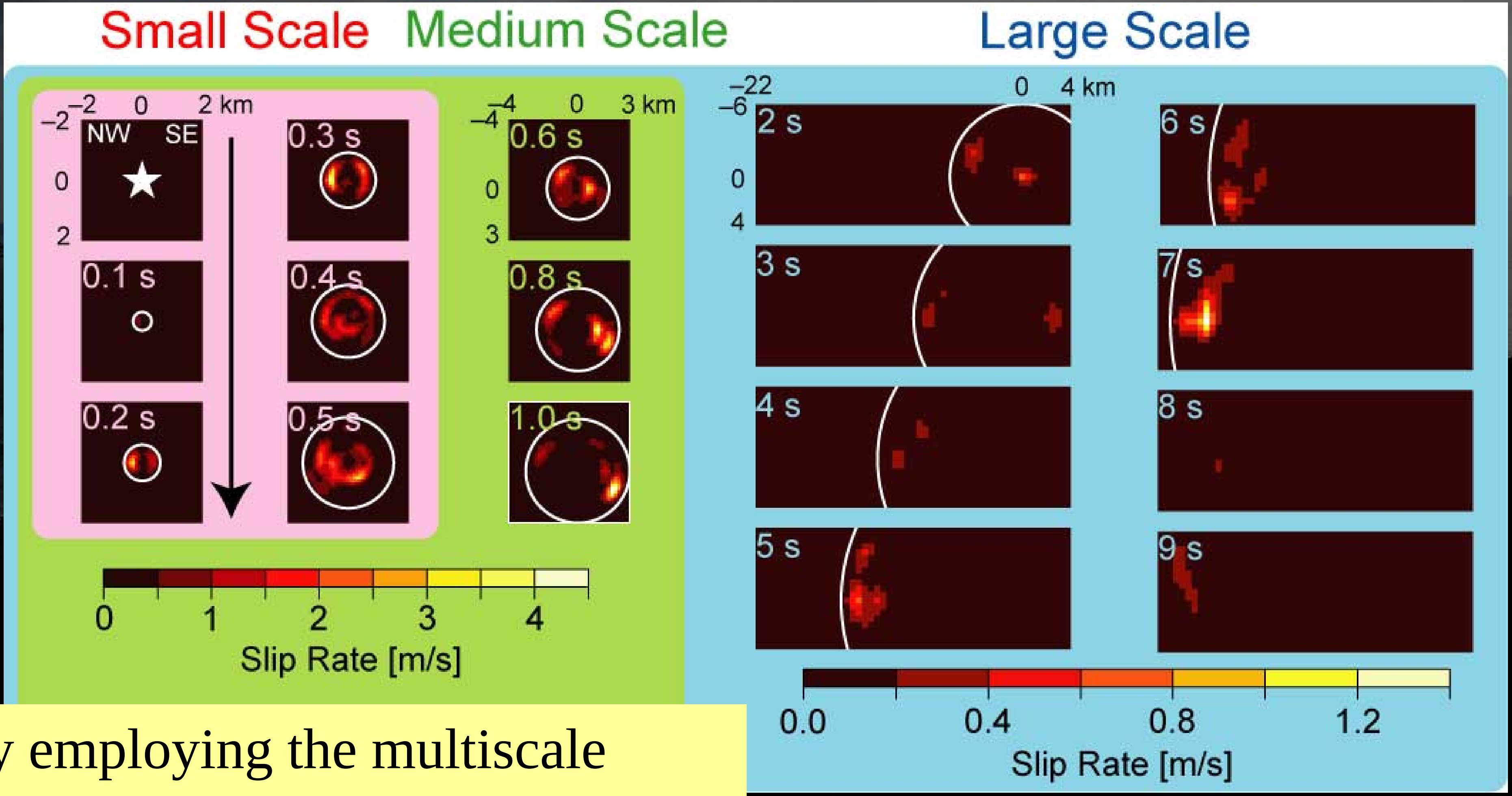
[4] Observation and Synthetic Waveforms



Reduced Variance: 68 %

The 2004 Parkfield Earthquake

[5] Snapshots



By employing the multiscale source model, the early stage of rupture is resolved well.

Slip is allowed within white circle
(Expanding speed: 3.0km/s)

Summary
Implication to SIV Project

- I have disclosed my inversion process in detail.
  - Multiscale source inversion.
  - How to decide the model assumptions.
  - How to determine the strength of the smoothing.
  - I prefer a smaller source model as possible.
- For inversion analyses, we give many and important assumptions. Check the assumptions.

- The resolution study is essential, though we are facing difficulties:
  - Covariance of data (due to limited frequency band and station locations).
    - Overcoming by appropriate resampling and station selections.
  - Error of Green's functions.

References

- AIC, ABIC
  - Akaike [1980]
  - Yabuki and Matsu'ura [GJI, 1992]
  - Yoshida et al. [PEPI, 1989]
  - Prof. Matsu'ura's lecture note (Univ. Tokyo, *in Japanese*)
- Multiscale Source Inversion
  - Uchide and Ide [JGR, 2007]
  - Uchide et al. [GRL, 2009]