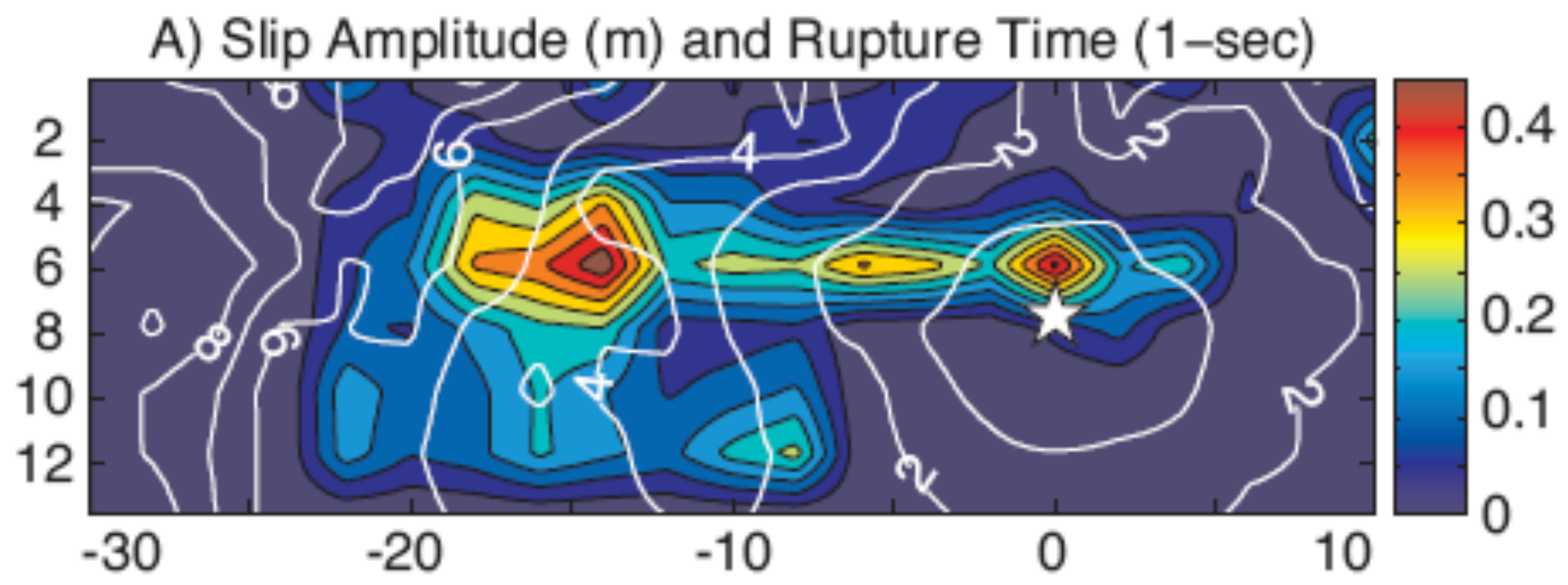
Strategies for Uncertainty Assessment in Source Inversions



Morgan Page

USGS Pasadena, formerly UC Santa Barbara

with

Susana Custódio, Ralph Archuleta, and Jean Carlson

UC Santa Barbara

Kinematic Inversions - Background

$$Ax = f$$

Medium response (Green's function)

Slip on fault

Data

Kinematic Inversions - Background

$$Ax = f$$

Medium response (Green's function)

Slip on fault

Data

$$\tilde{x} = \tilde{A}f = \tilde{A}(Ax) = (\tilde{A}A)x$$

Estimated Model solution

GI

True solution

The minimum-length least-squares solution is found using the Moore-Penrose Generalized Inverse (GI).

Kinematic Inversions - Background

$$Ax = f$$

Medium response (Green's function)

Slip on fault

Data

$$\tilde{x} = \tilde{A}f = \tilde{A}(Ax) = (\tilde{A}A)x$$

Estimated Model solution

GI

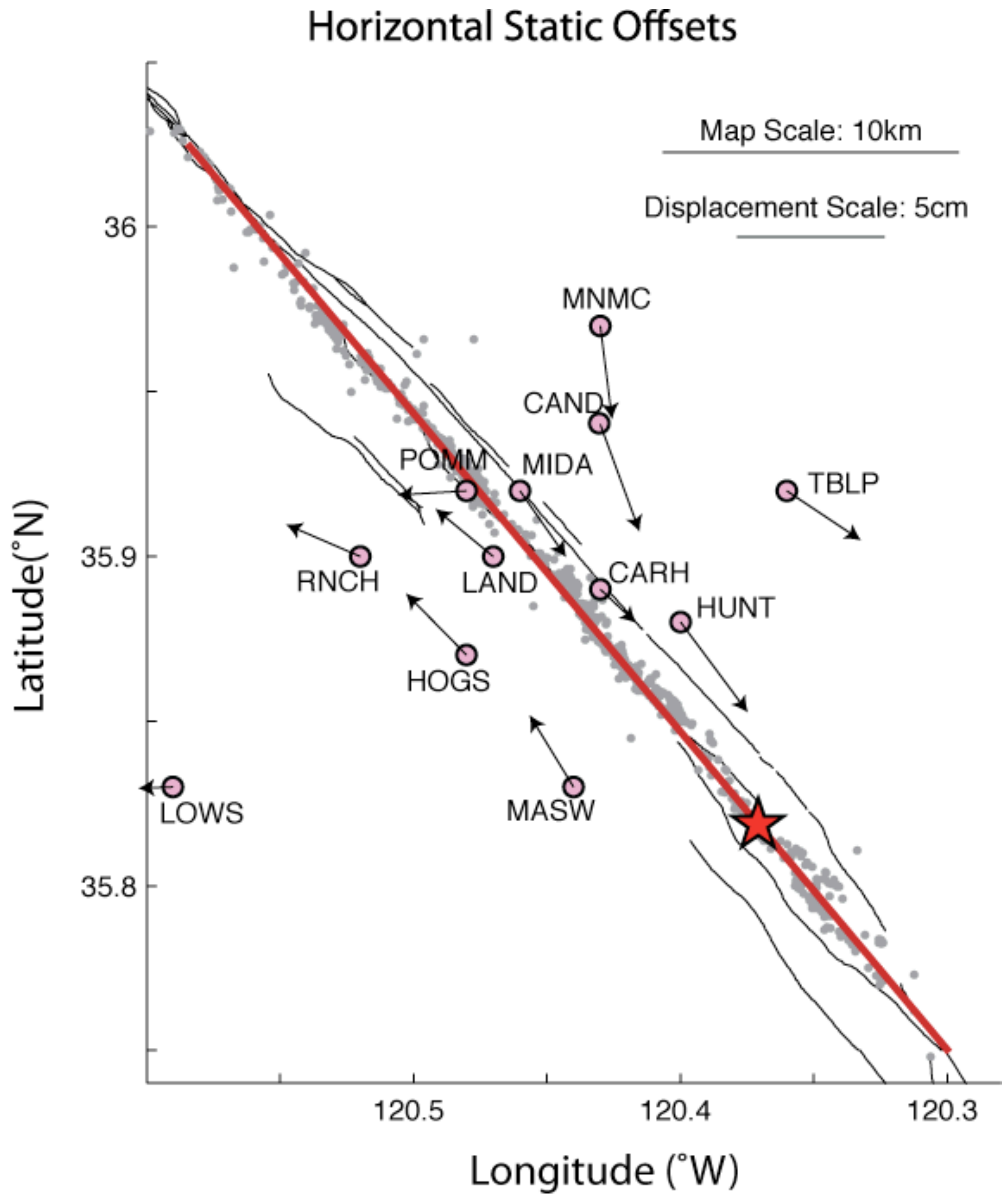
True solution

Resolution matrix

$$R = \tilde{A}A$$

The resolution matrix contains information about tradeoffs between model parameters that the available data cannot resolve.

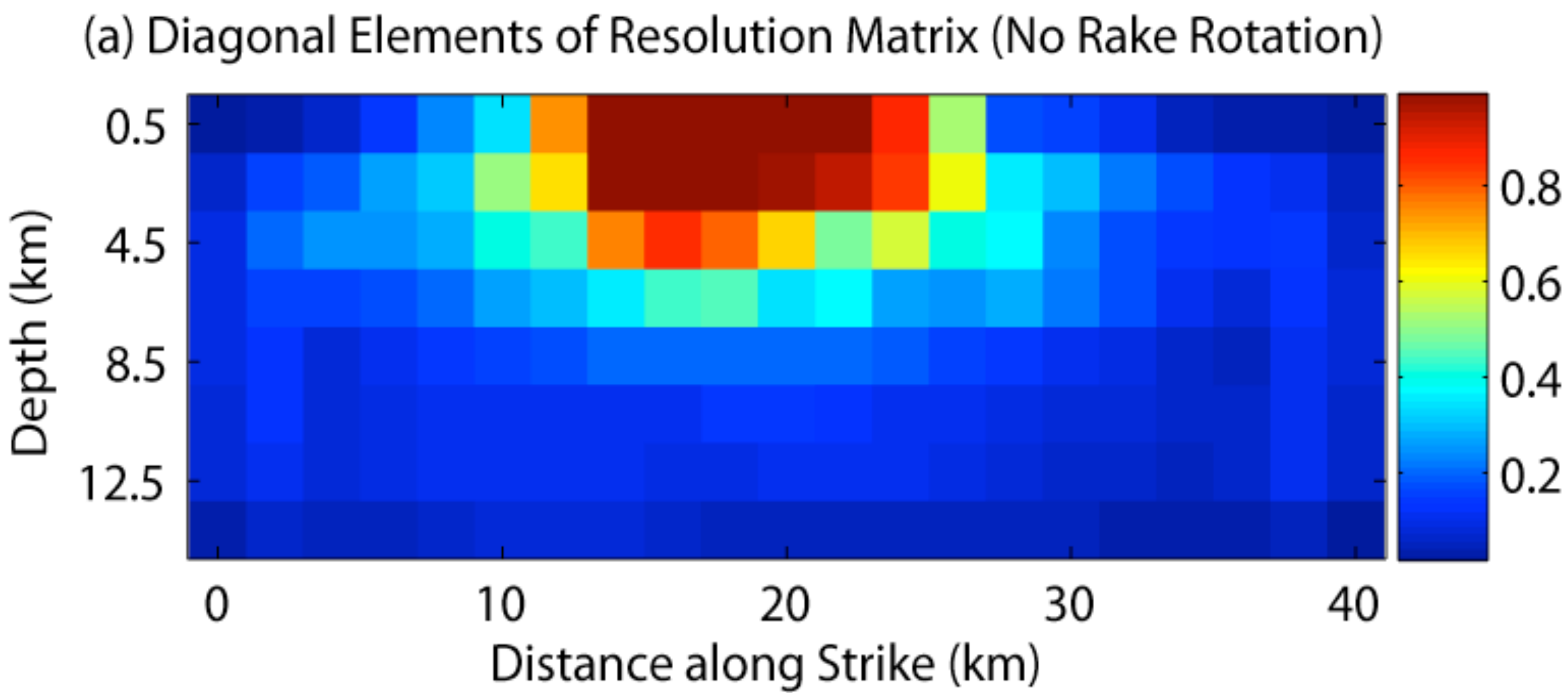
2004 M_w6.0 Parkfield Earthquake: Inversion of GPS Data



Model assumptions:

- ✓ The fault strikes 140° SE and dips 87° SW;
- ✓ The rupture was at maximum 40 km long;
- ✓ The ruptured area is deeper than 0.5 km (no surface rupture);
- ✓ The velocity structure is well approximated by a 1D bilateral model (NE - slow; SW - fast; after Eberhart-Phillips and Michael (1993) and Thurber et al. (2003)).

The Parkfield GPS Resolution Matrix Visualized



model solution (inversion result)

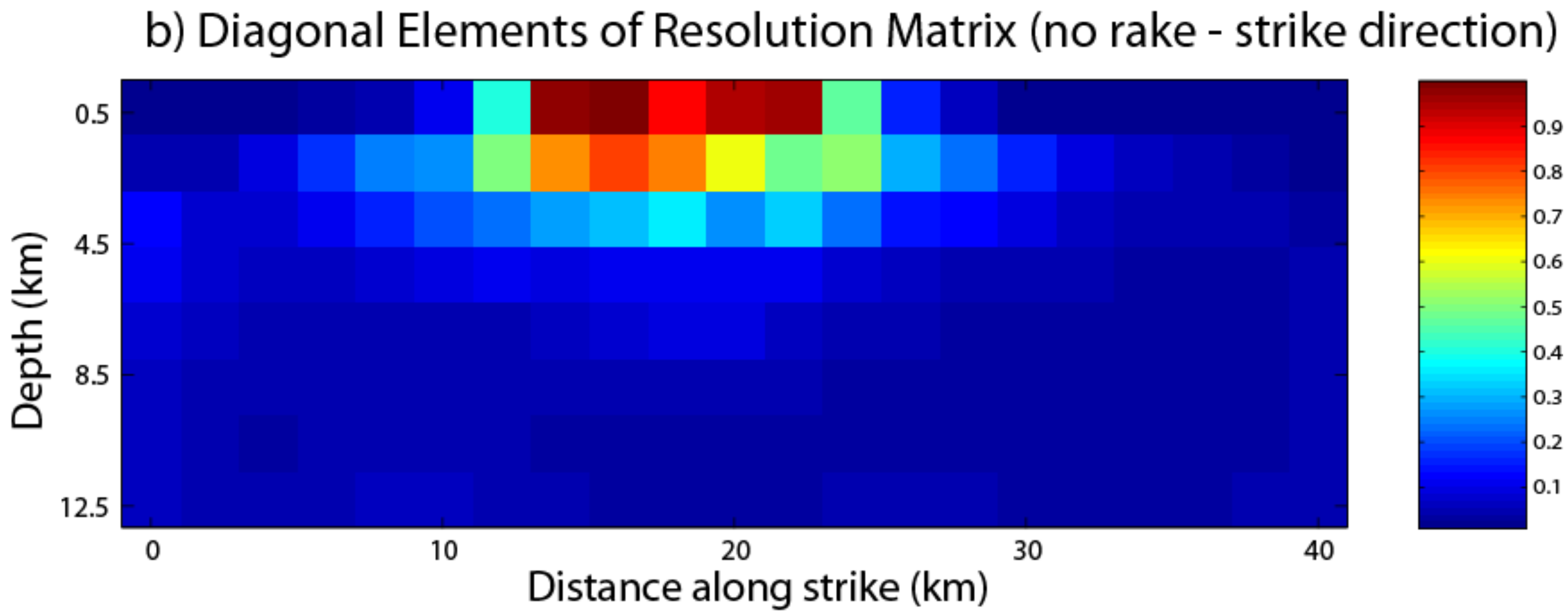
true solution (unknown)

$$\tilde{x} = \tilde{A}f = \tilde{A}(Ax) = (\tilde{A}A)x$$

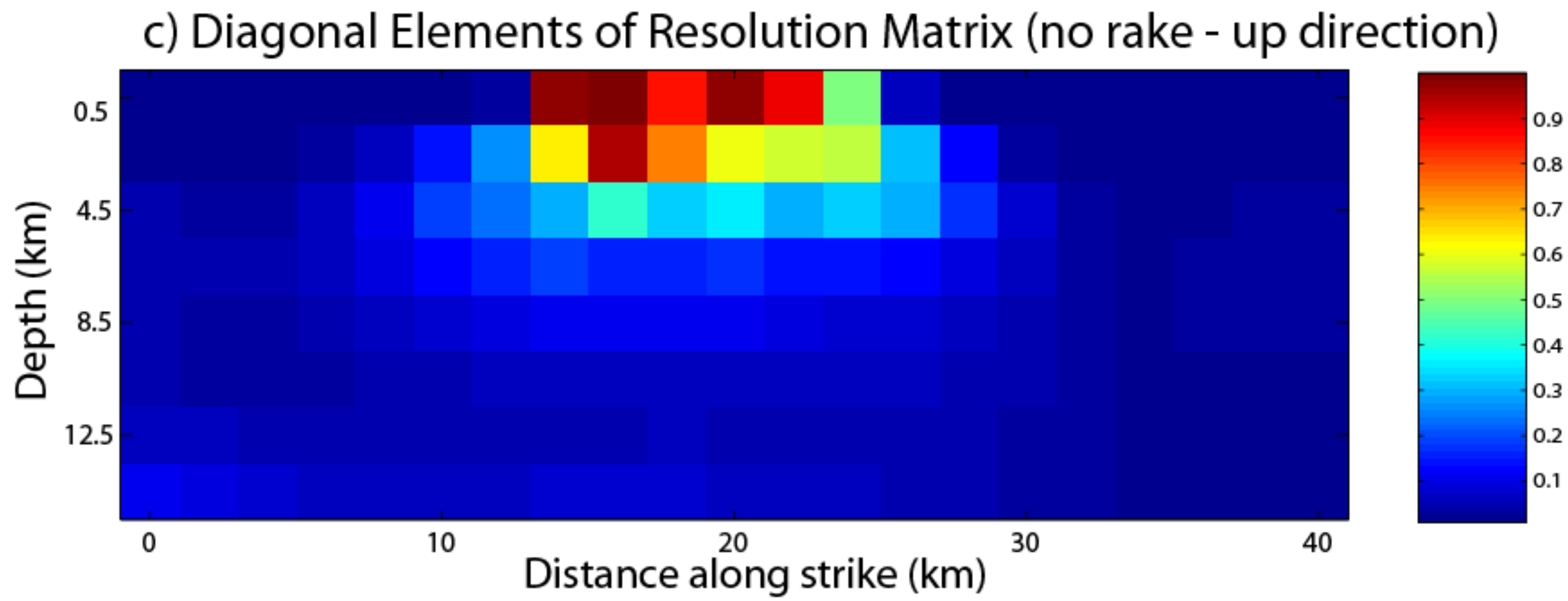
Resolution matrix $R = \tilde{A}A$

The resolution matrix contains information about tradeoffs between model parameters that the available data cannot resolve.

Diagonal elements of Resolution Matrix (with rake)



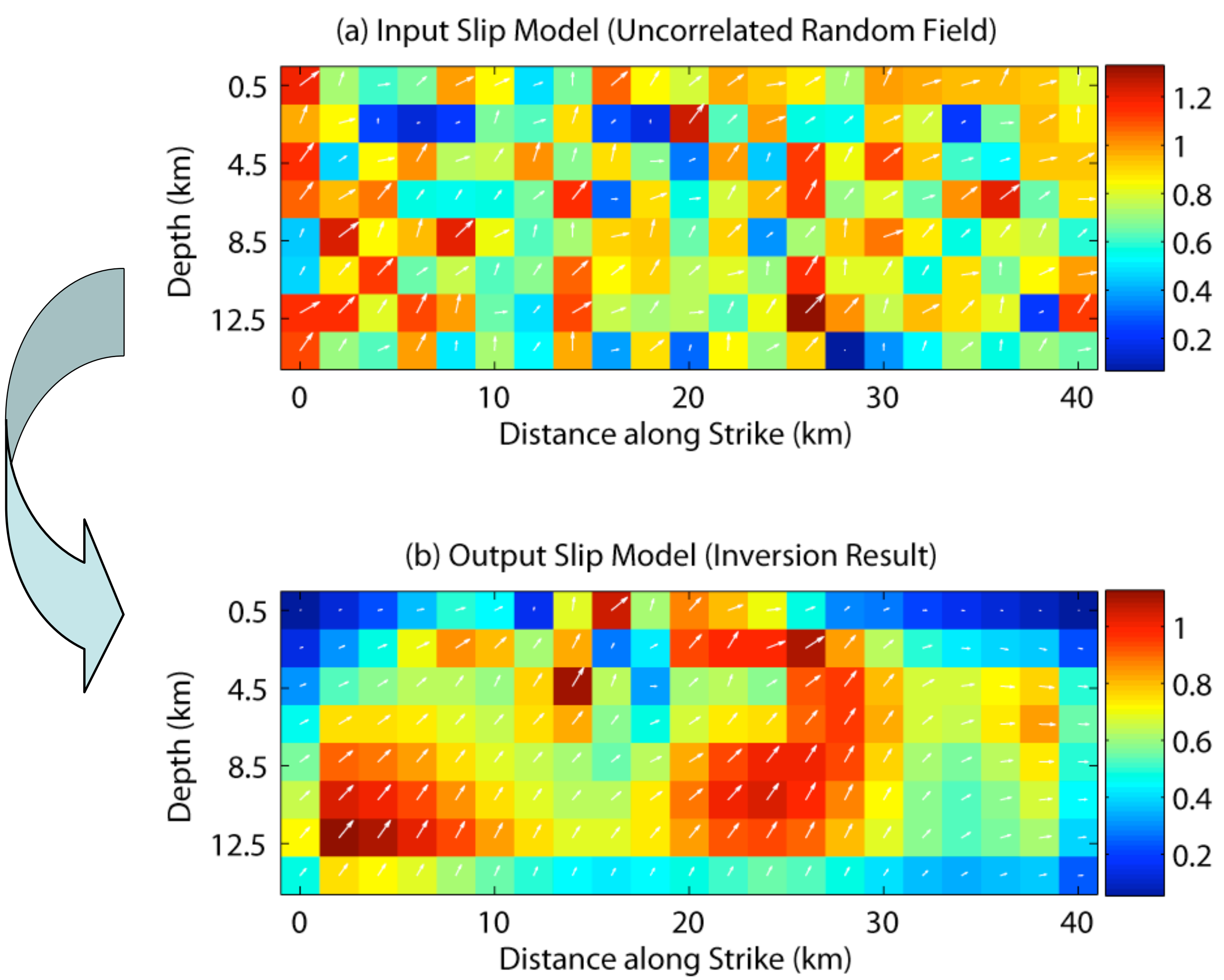
strike direction



up direction

The importance of the off-diagonal elements of R

Due to structure in R, it is possible to get structure by inverting a random field

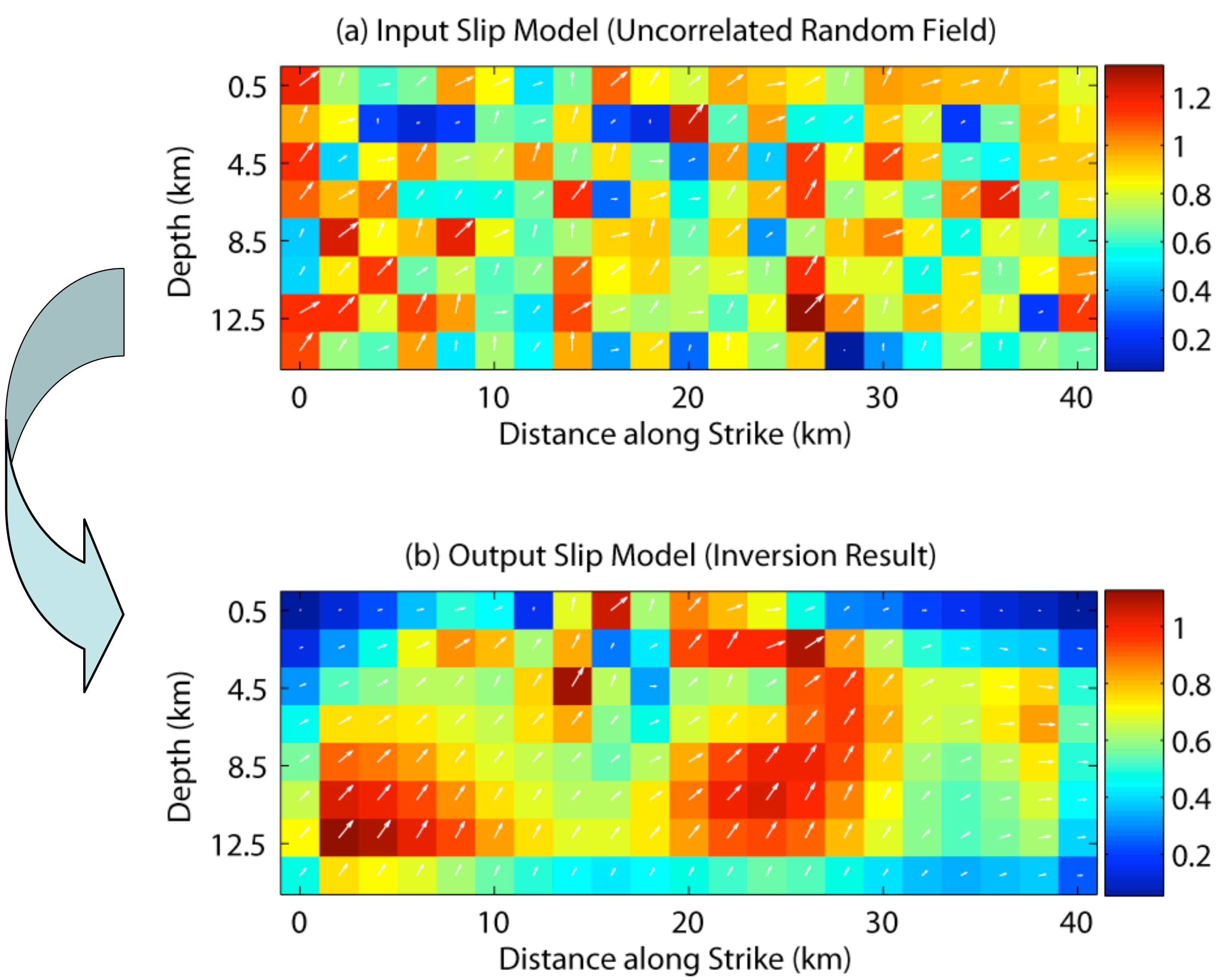


input model
(synthetic)

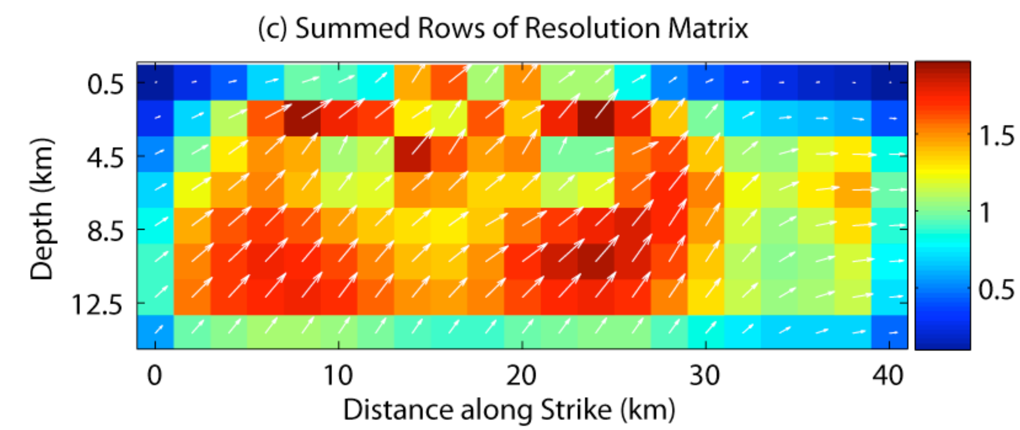
output
(inversion result)

The importance of the off-diagonal elements of R

Due to structure in R, it is possible to get structure by inverting a random field

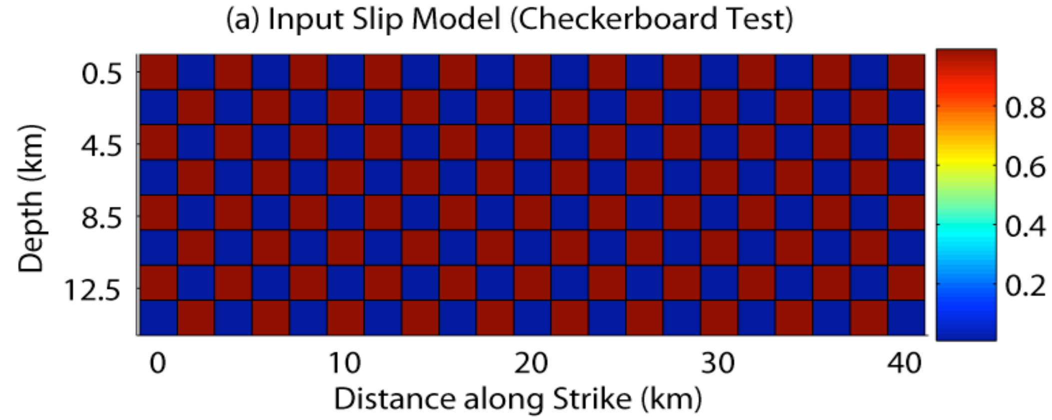


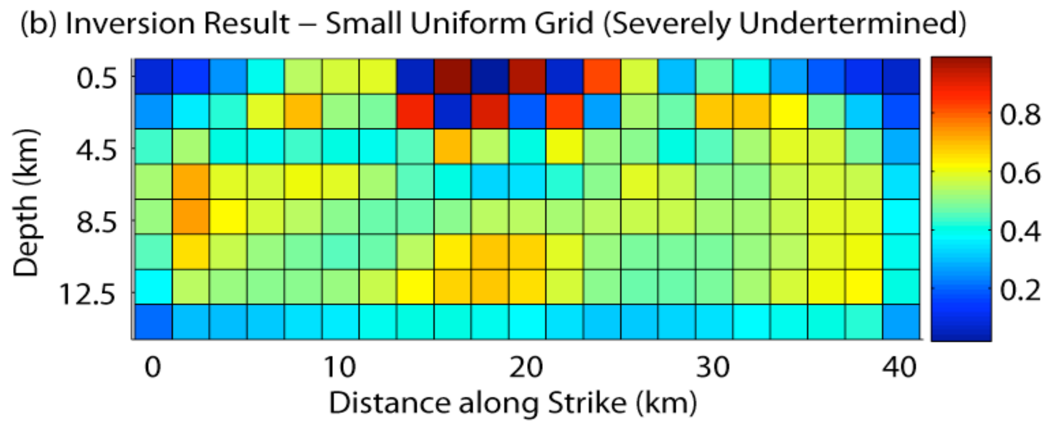
Summed Rows of R
(with rake)



Alternative Griddings, with Resolution in Mind

Input Model (same for each grid)



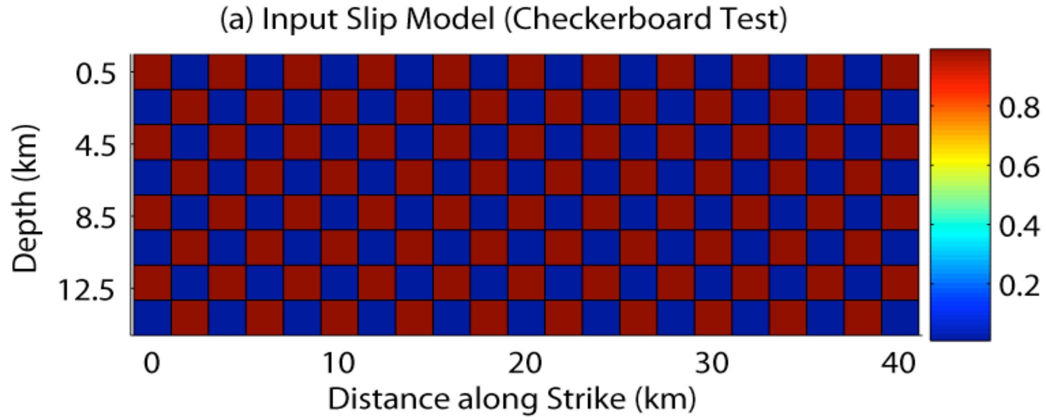


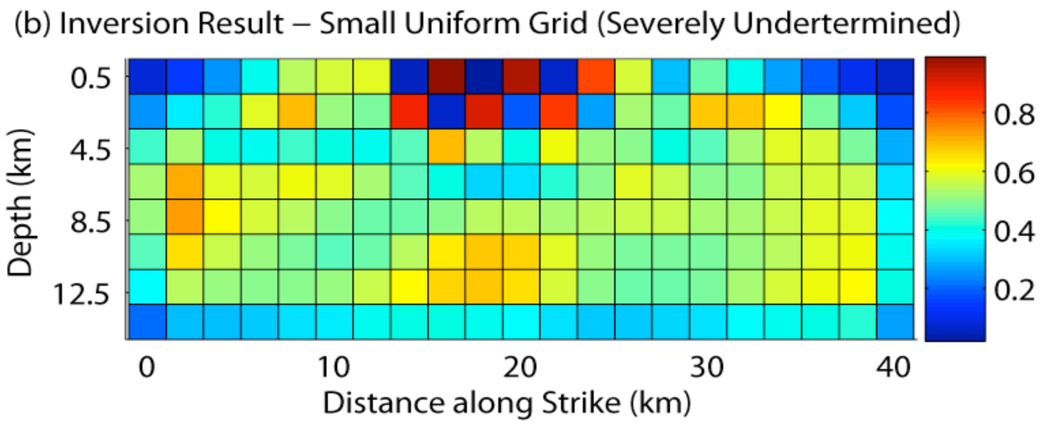
Small uniform grid: Correctly recovers structure near surface, but generates spurious structure in poorly resolved areas

Smoothing can get rid of the spurious structure at depth, but some structure near the surface is lost.

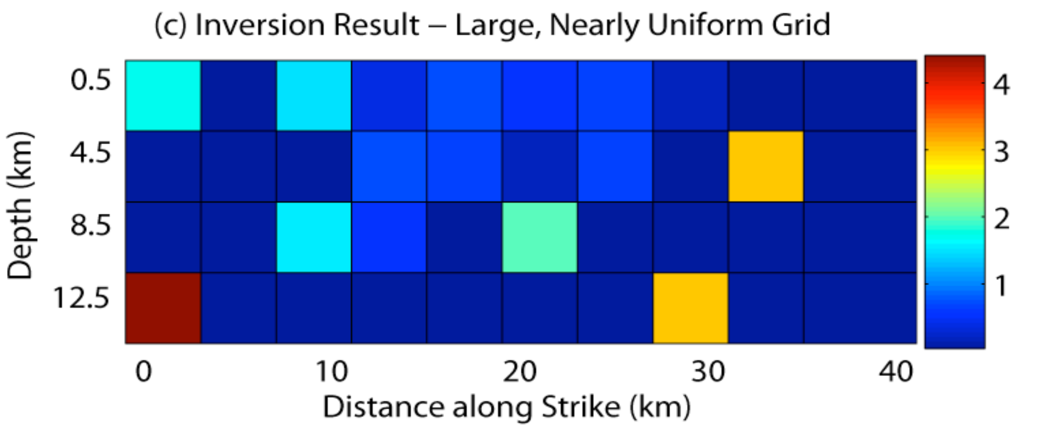
Alternative Griddings, with Resolution in Mind

Input Model (same for each grid)





Small uniform grid: Correctly recovers structure near surface, but generates spurious structure in poorly resolved areas

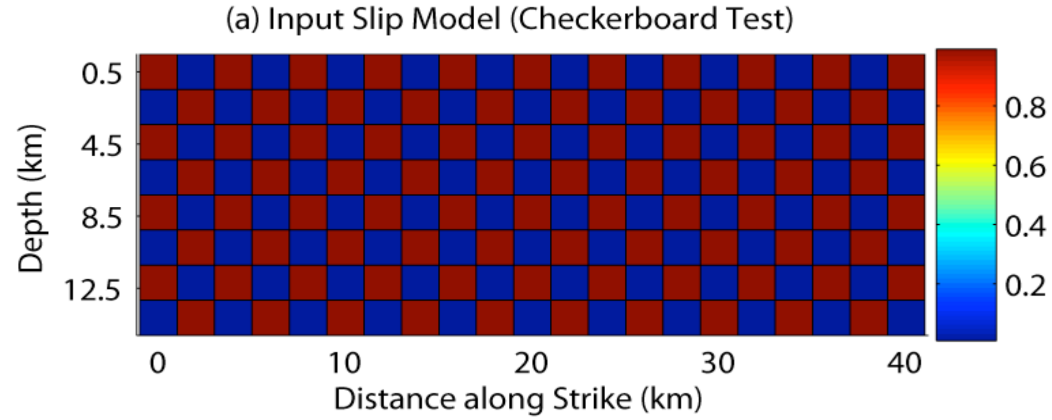


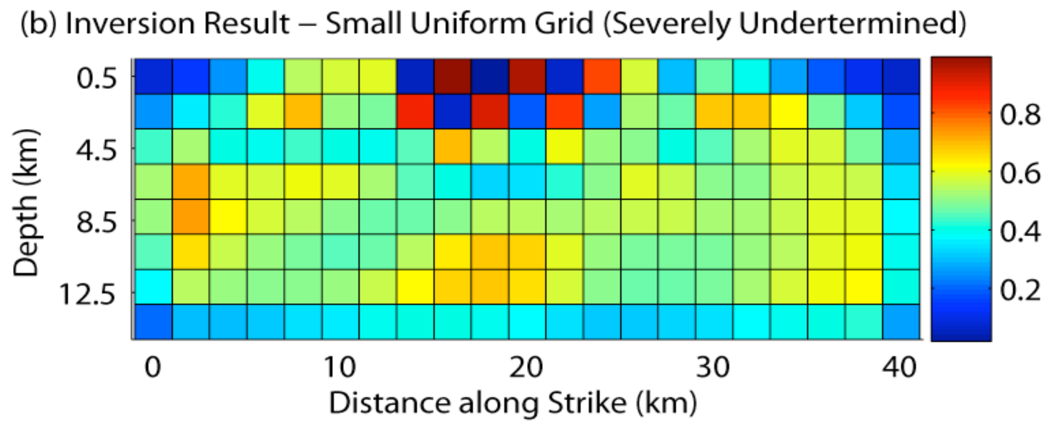
Large uniform grid: Loses information near surface, gives incorrect slip at depth

(near the surface the inversion is sensitive to structure on a finer scale than is parameterized)

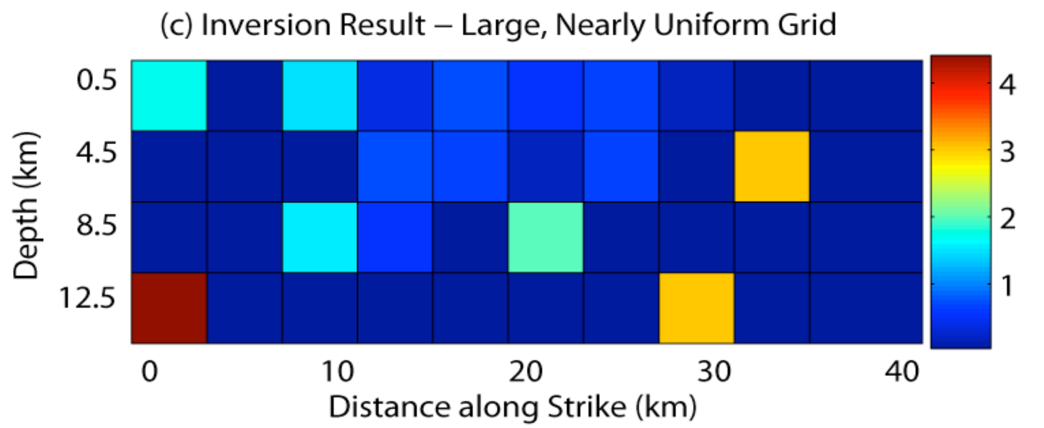
Alternative Griddings, with Resolution in Mind

Input Model (same for each grid)

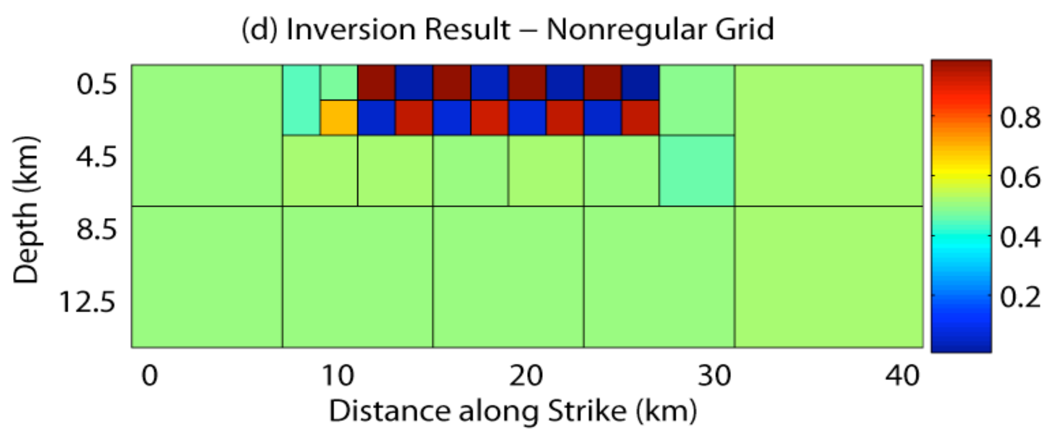




Small uniform grid: Correctly recovers structure near surface, but generates spurious structure in poorly resolved areas



Large uniform grid: Loses information near surface, gives incorrect slip at depth



Non-regular grid: Correctly recovers structure near surface, and averages out slip correctly at depth

Nonuniform smoothing can achieve a similar result!

Two types of error must be parameterized in order to interpret inversion results:

Resolution error *(error due to "under-determined-ness")*

- Technically unbounded!
- Resolution matrix gives length scales

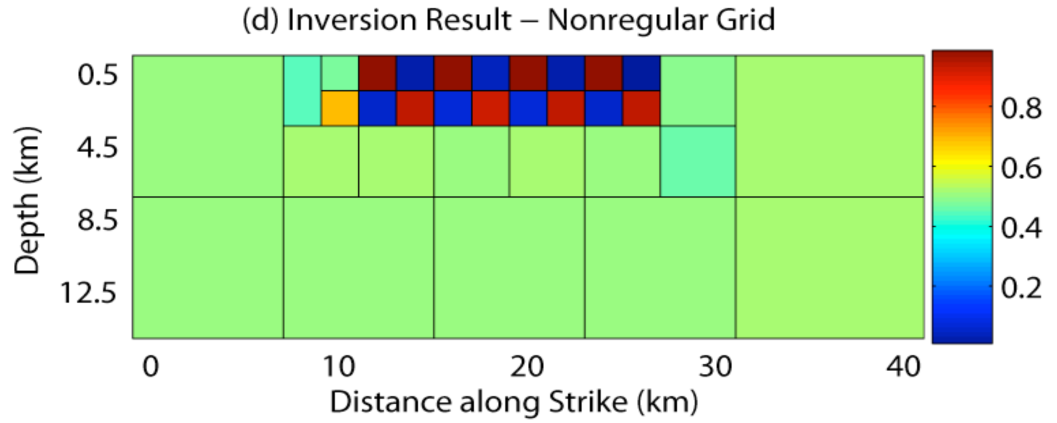
Perturbation error *(error due to Green's function & data errors)*

- Can easily be sampled with Monte Carlo sampling
- Model Covariance Matrix gives error bounds

Advantages of nonuniform grid:

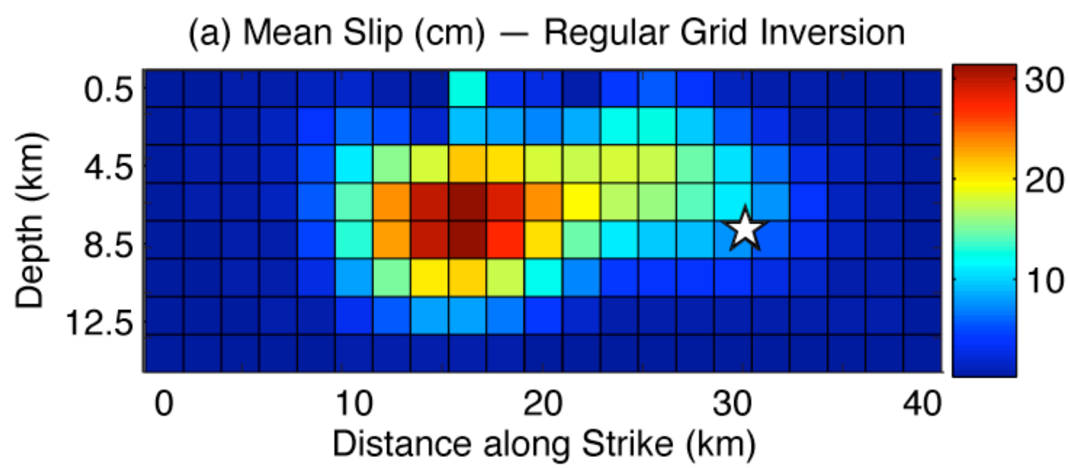
✓ Resolution error & perturbation error easily separated and easy to visualize

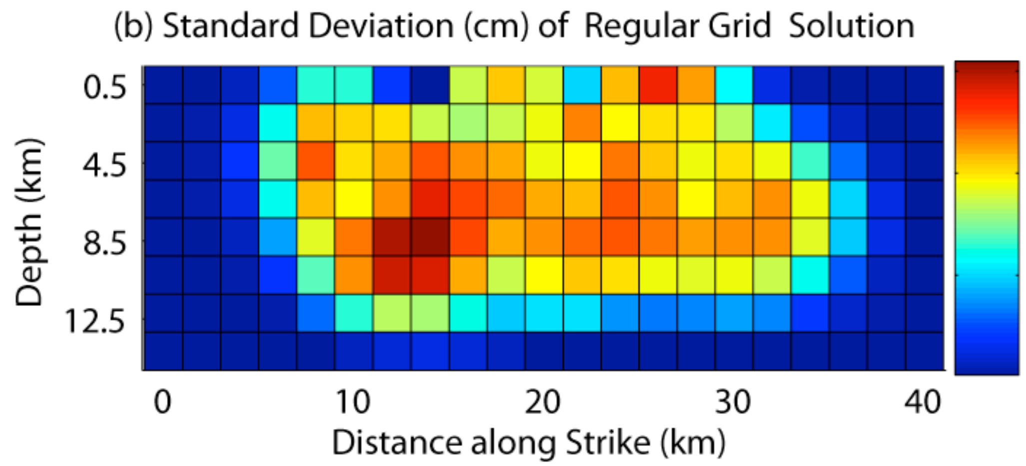
✓ Structure in well-resolved areas recovered while spurious structure is avoided



Inversion of Parkfield GPS Data

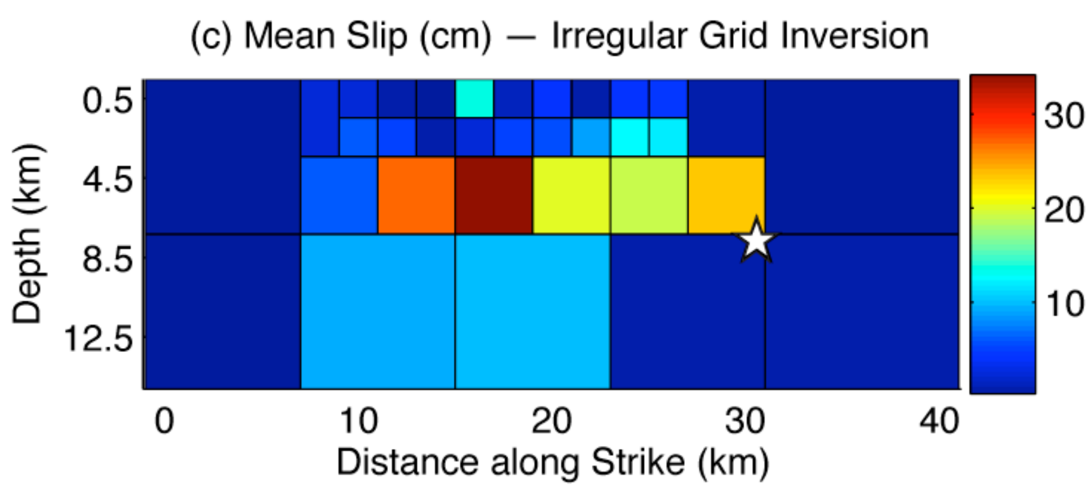
On Regular Grid:

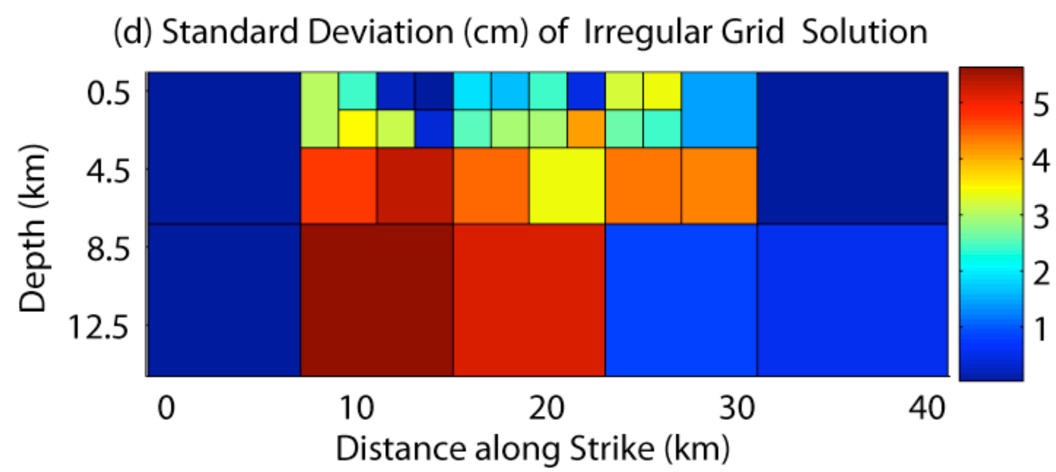




Data errors sampled via Monte Carlo

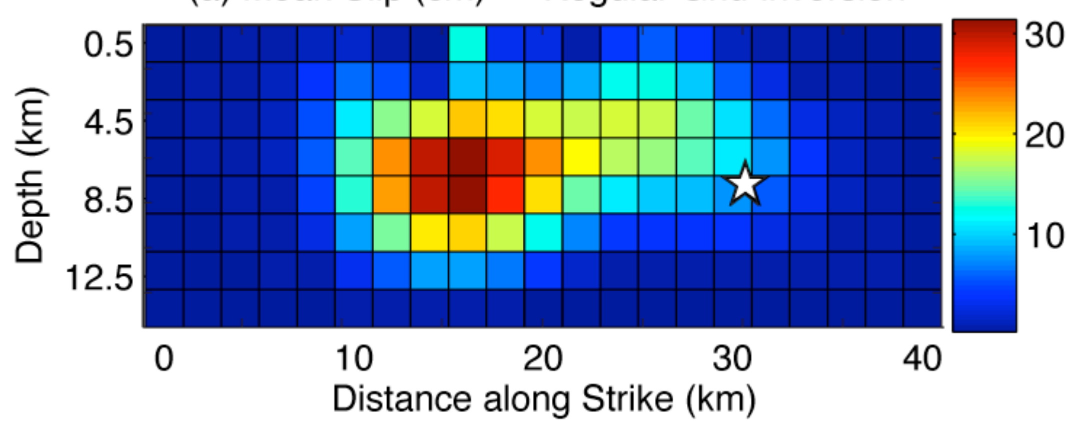
On Irregular Grid:



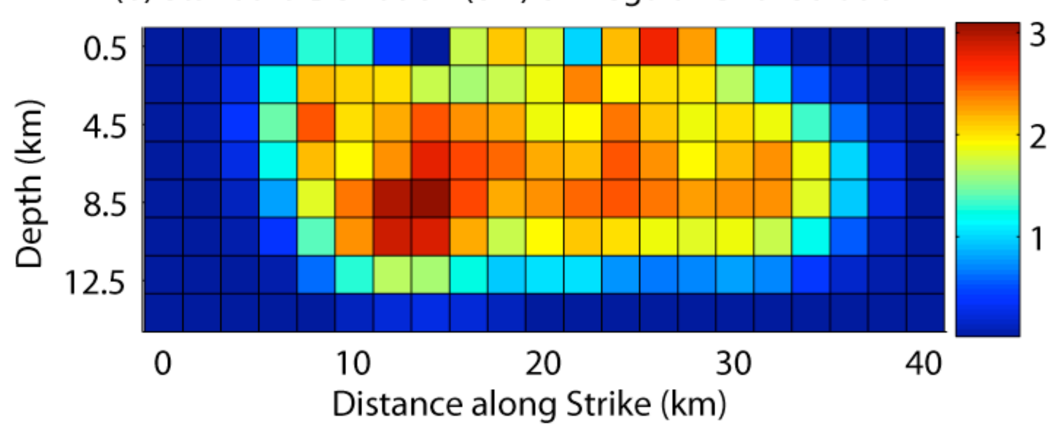


Inversion of Parkfield GPS Data

On Regular Grid:



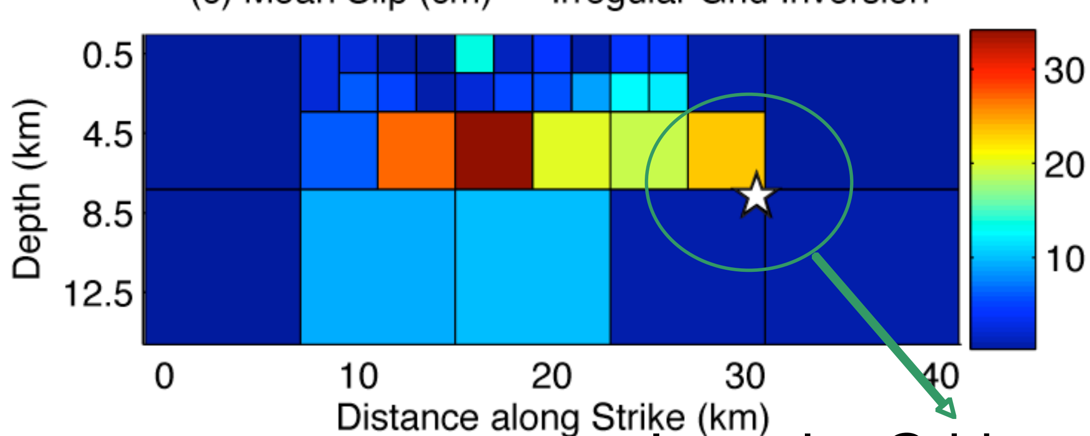
(a) Mean Slip (cm) — Regular Grid Inversion



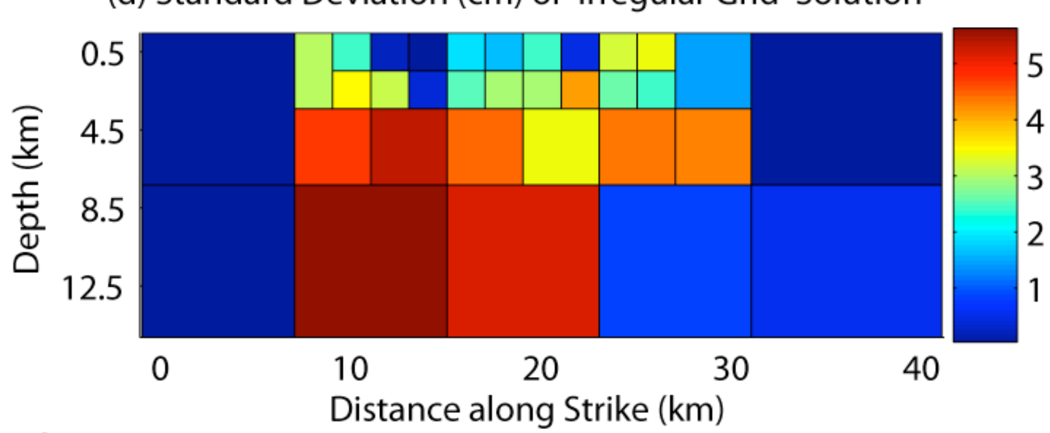
(b) Standard Deviation (cm) of Regular Grid Solution

Data errors sampled via Monte Carlo

On Irregular Grid:



(c) Mean Slip (cm) — Irregular Grid Inversion



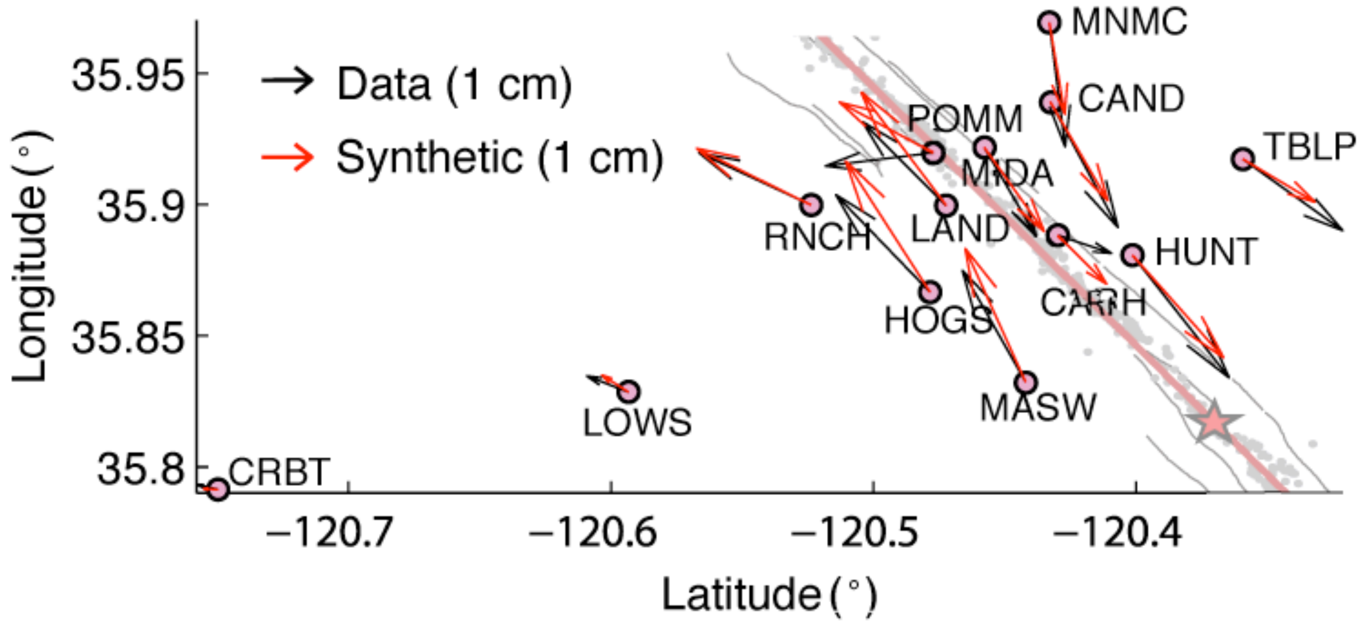
(d) Standard Deviation (cm) of Irregular Grid Solution

Irregular Grid results in
more slip near hypocenter

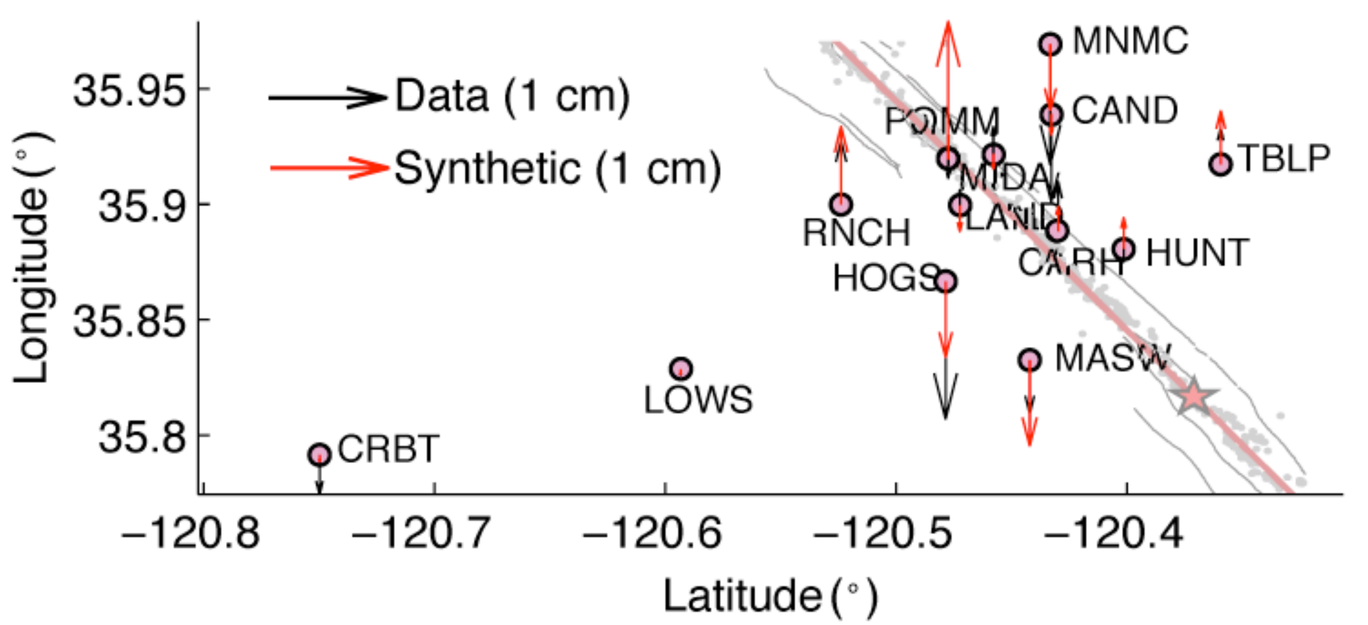
Match to Data

Irregular Grid – Variance Reduction of 89%

(e) Irregular Grid Inversion Data Fit -- Horizontal Component

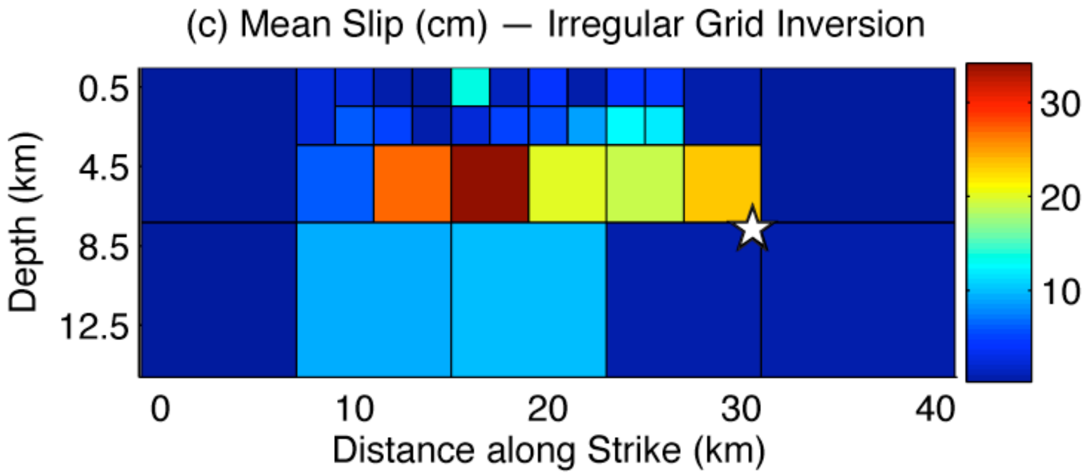


(f) Irregular Grid Inversion Data Fit -- Vertical Component

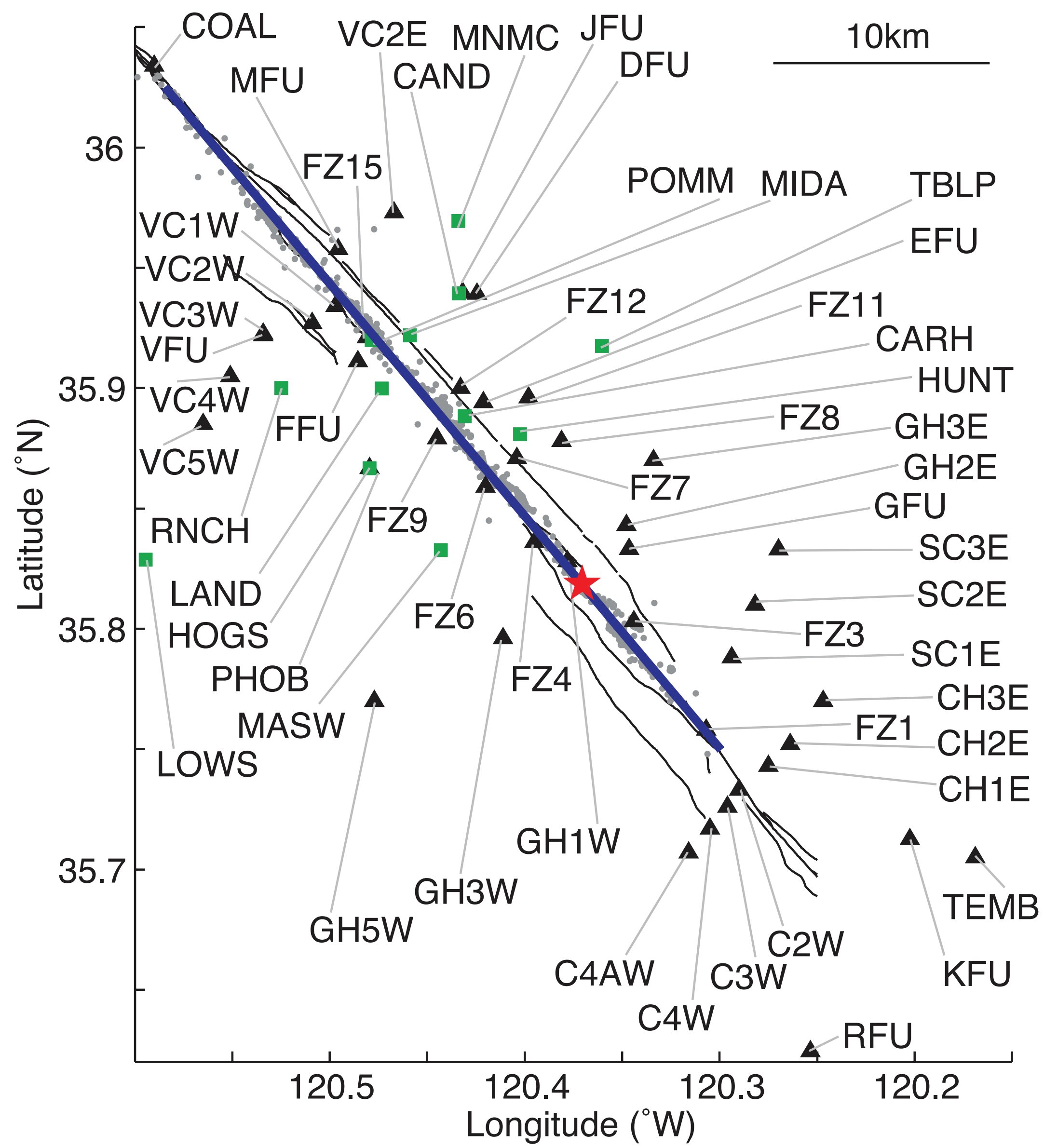


Strong-motion Inversion
constrained by GPS slip model

Irregular Grid:

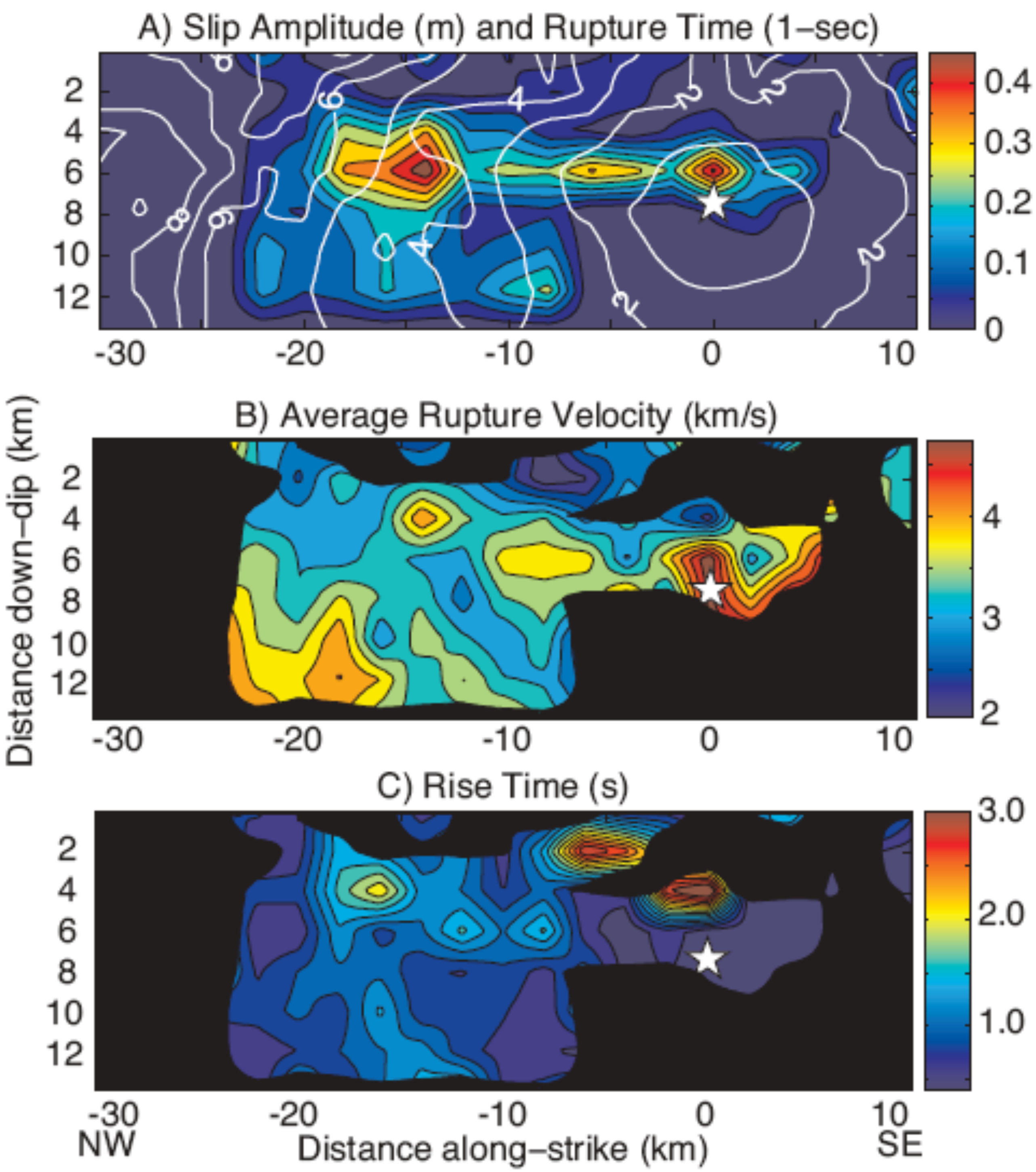


+

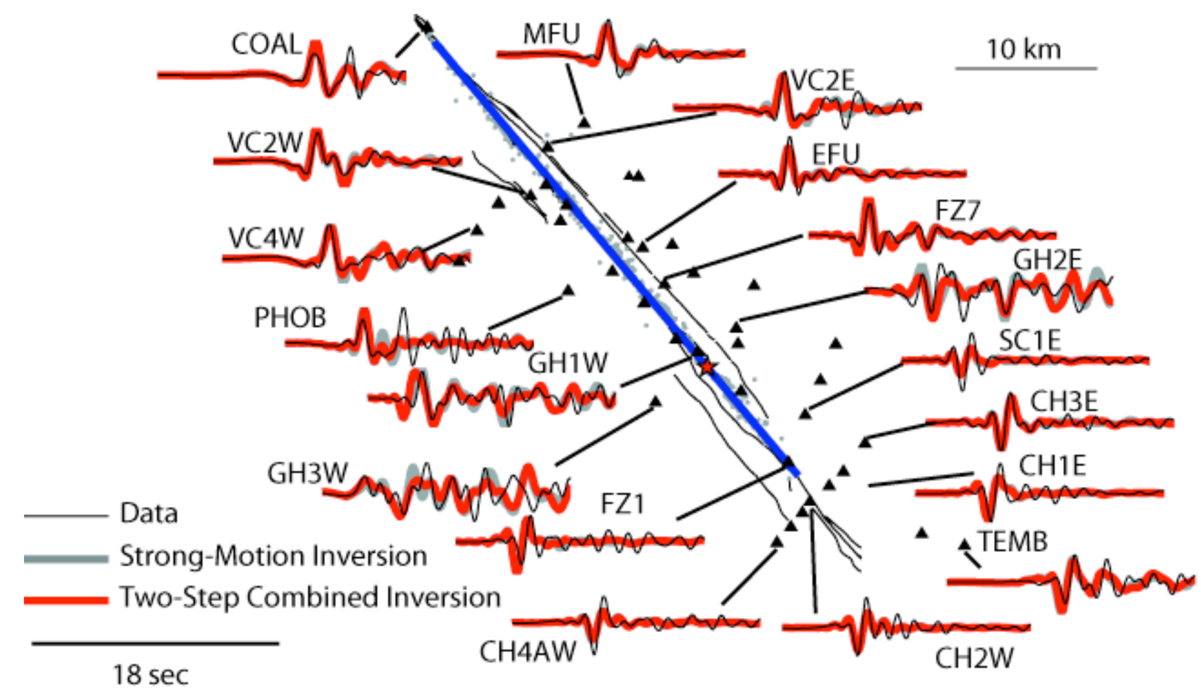


We want to constrain the final slip in the strong-motion inversion to match our GPS inversion within the error bounds.

Two-step inversion of GPS and strong-motion data

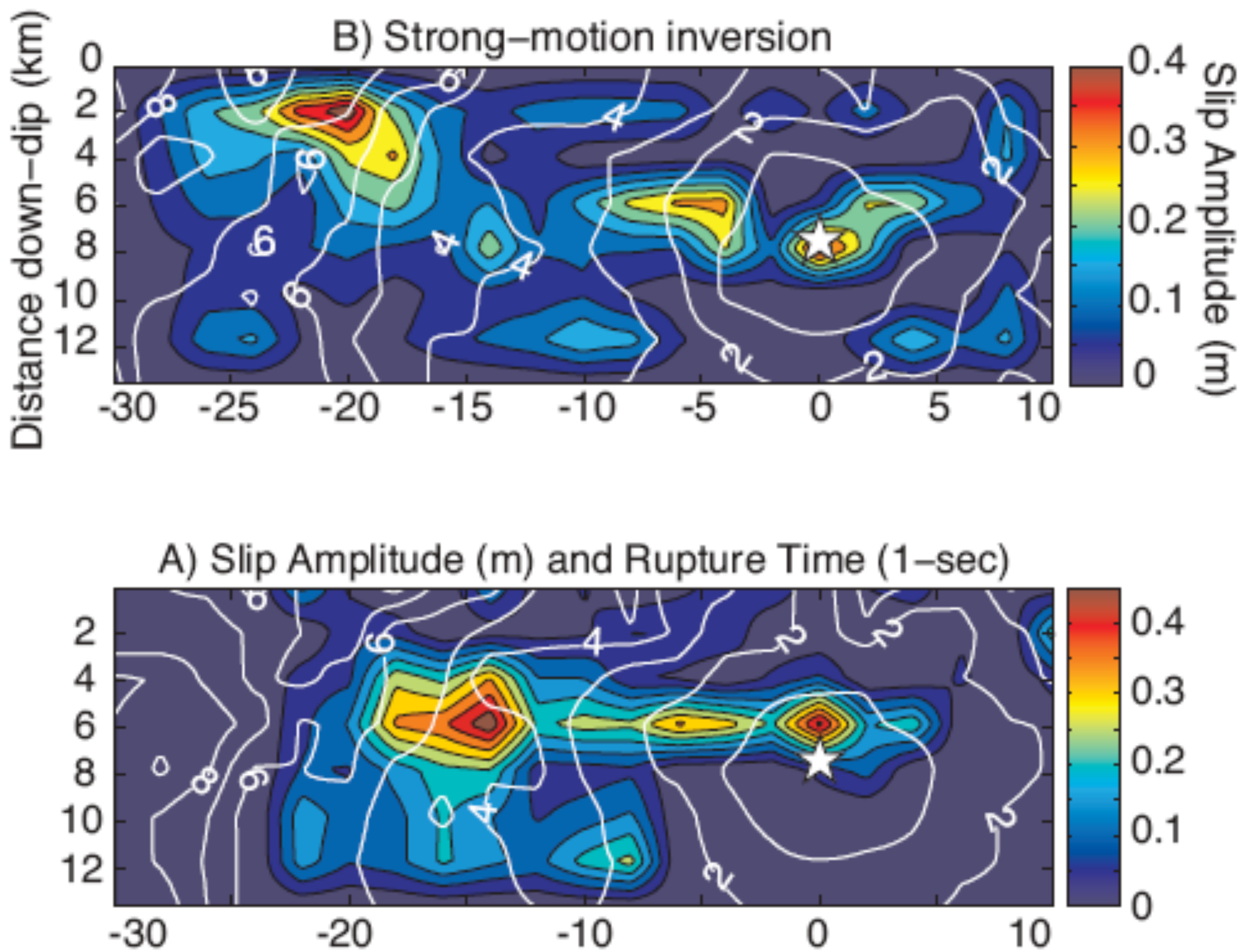


Preferred slip model



Slip near hypocenter required to fit stations to the southeast

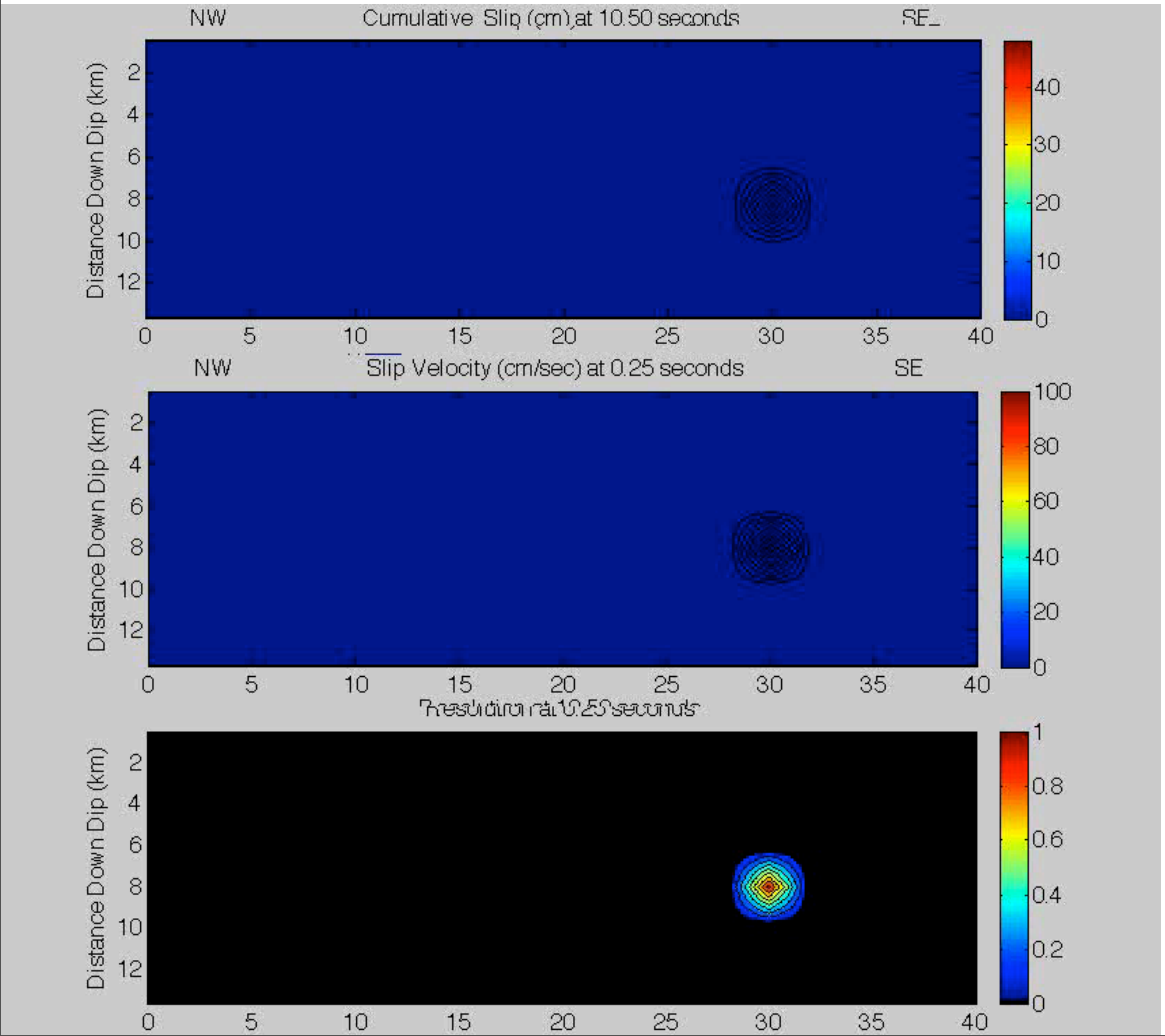
How the addition of GPS data changes the result



Strong motion only

GPS + strong motion

Resolution for Strong-Motion Inversion (movie)

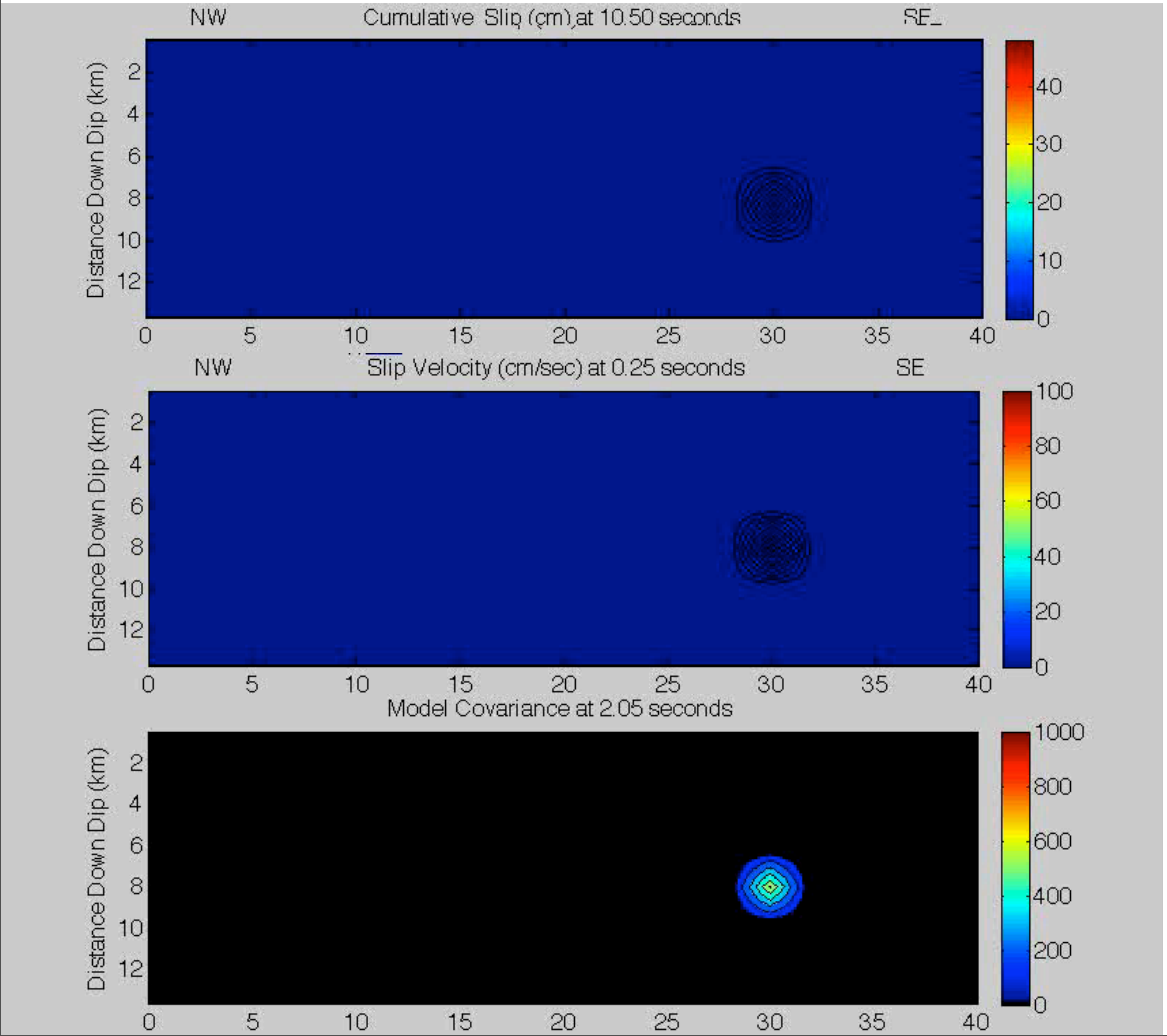


Cumulative Slip

Slip Velocity

Resolution

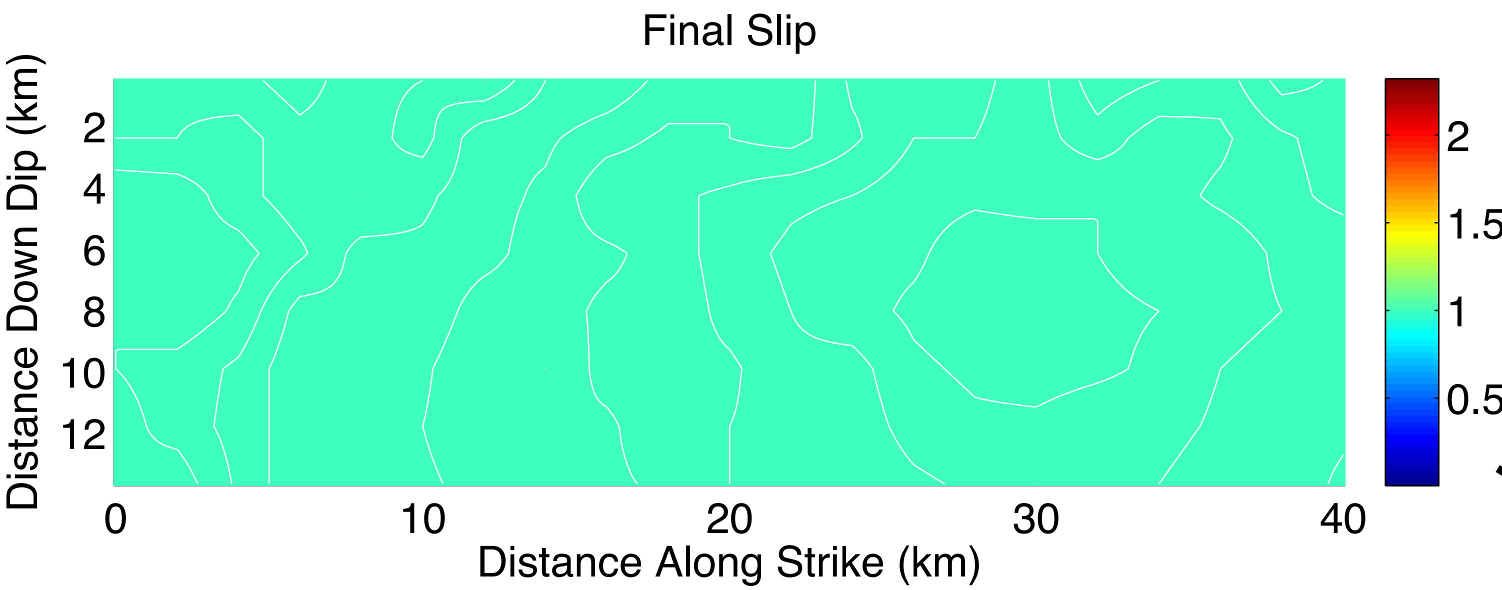
Model Covariance for Strong-Motion Inversion (movie)

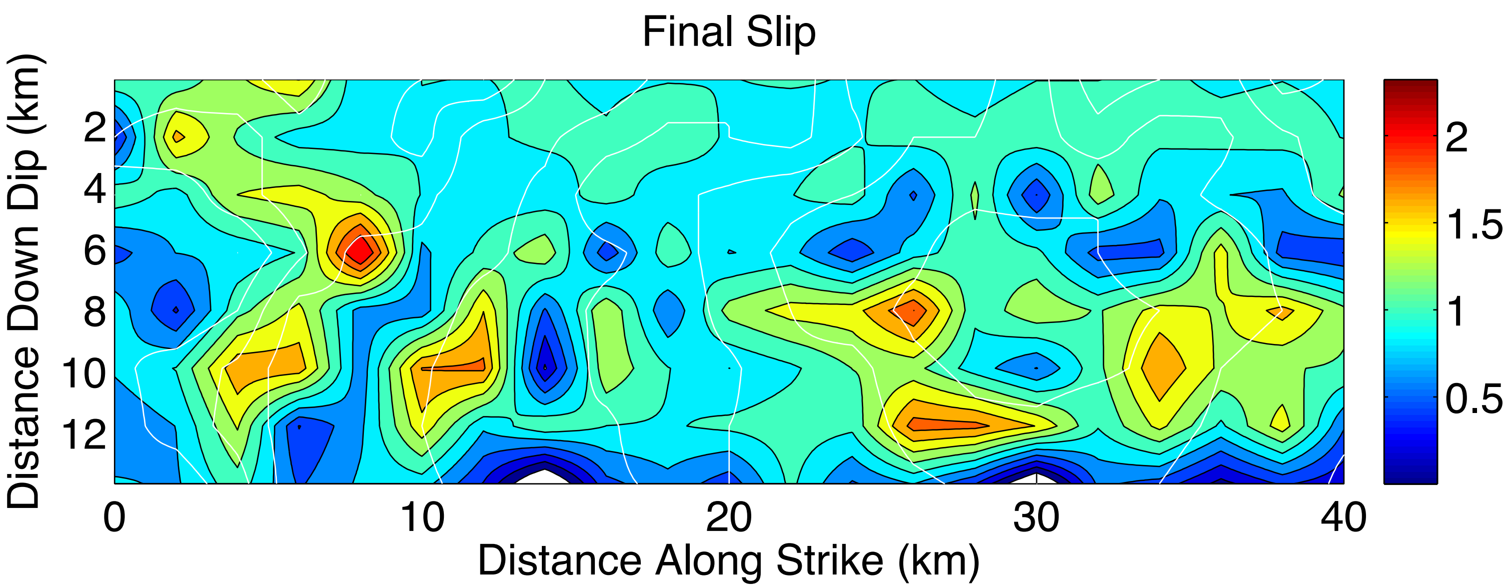


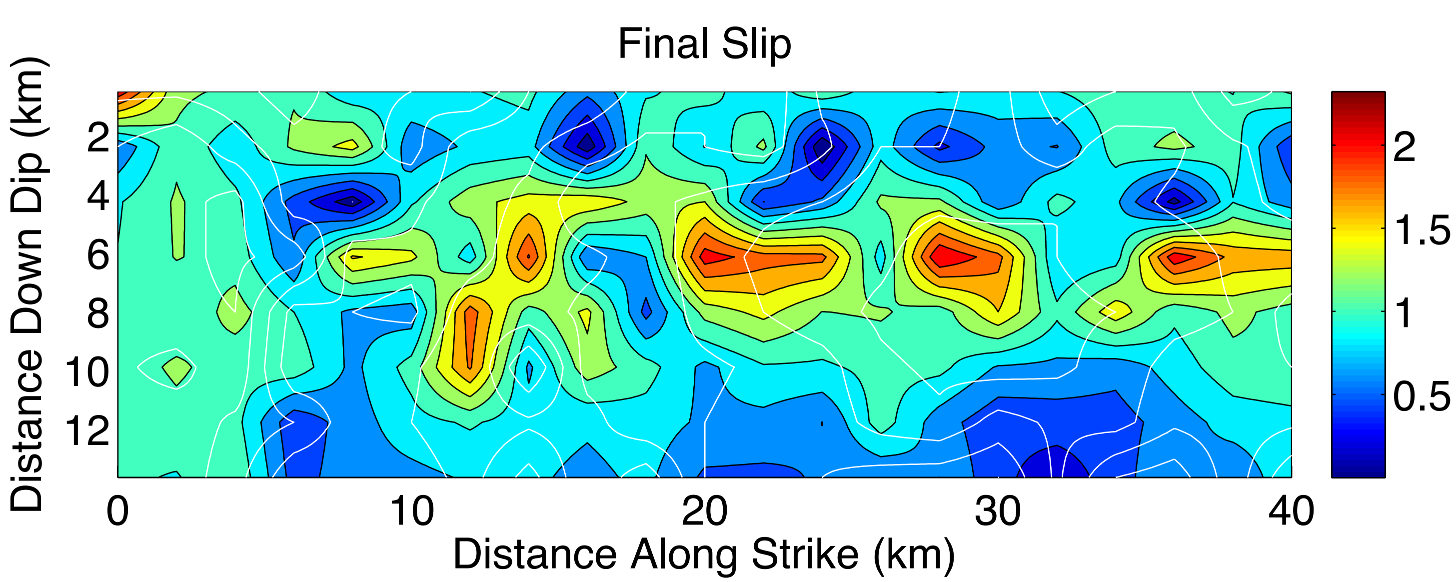
Cumulative Slip

Slip Velocity

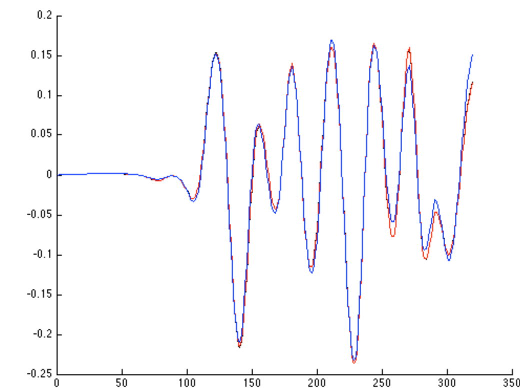
Covariance







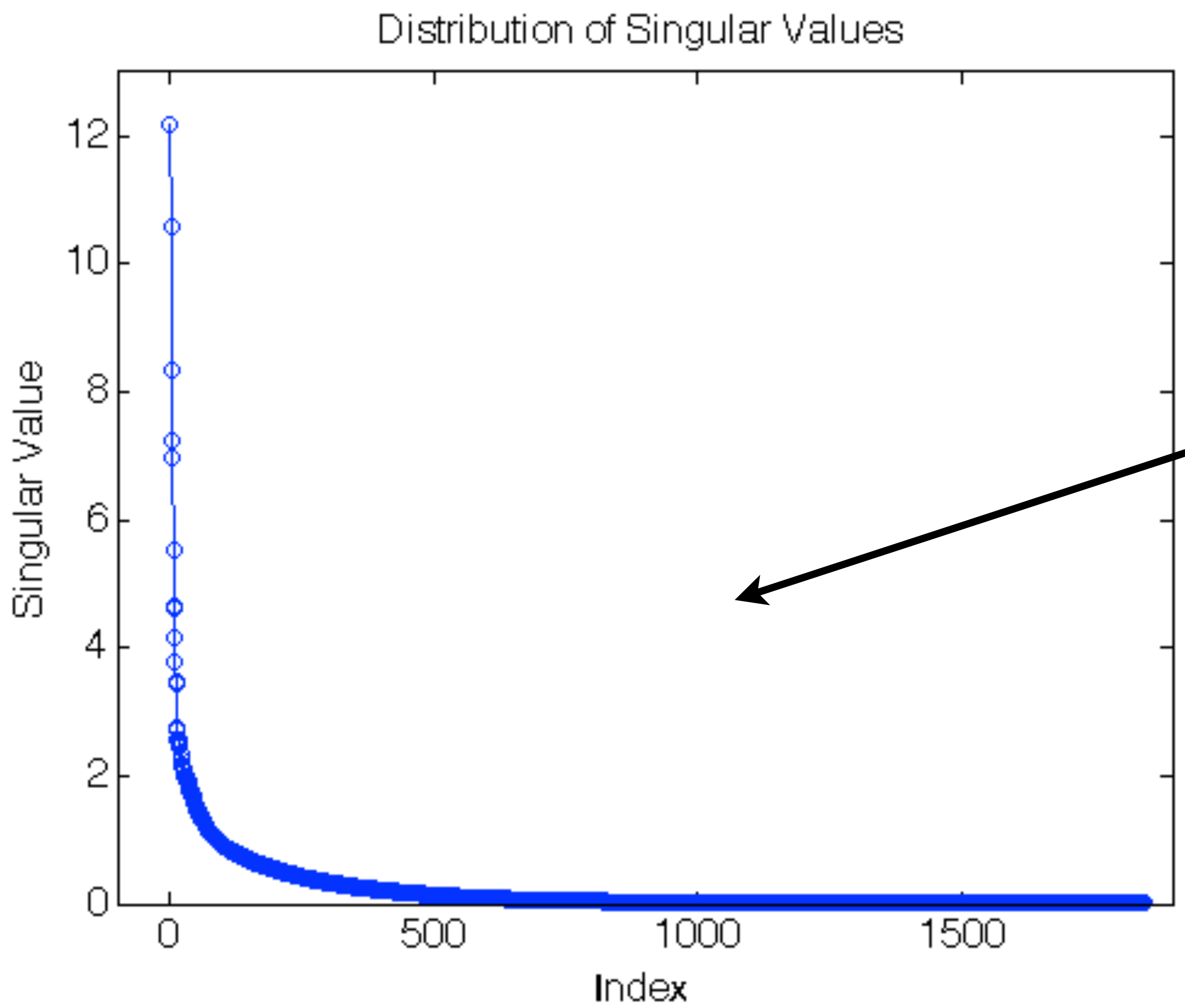
3 very different ruptures



- have equal moment
- generate identical seismograms at all 43 strong-motion stations

Inversion is ill-conditioned

Most data variance can be reproduced with only a few model parameters



Can somewhat optimize station distribution to improve condition number (e.g. fault-parallel arrays perform better than fault-crossing arrays)

Still, adding more stations does not fundamentally change the shape of the singular-value distribution

Strategies for Uncertainty Assessment

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**
  - Vary amount of smoothing or damping, add in null vectors to generate additional models, change model parameterization

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**
  - Vary amount of smoothing or damping, add in null vectors to generate additional models, change model parameterization
  - Present end-member models (e.g. smoothest model, minimum-moment model, GI solution)

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**
    - Vary amount of smoothing or damping, add in null vectors to generate additional models, change model parameterization
    - Present end-member models (e.g. smoothest model, minimum-moment model, GI solution)
    - Many nonlinear algorithms give multiple models (different random seed)

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**
  - Vary amount of smoothing or damping, add in null vectors to generate additional models, change model parameterization
  - Present end-member models (e.g. smoothest model, minimum-moment model, GI solution)
  - Many nonlinear algorithms give multiple models (different random seed)
- **Synthetic Tests**

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**
  - Vary amount of smoothing or damping, add in null vectors to generate additional models, change model parameterization
  - Present end-member models (e.g. smoothest model, minimum-moment model, GI solution)
  - Many nonlinear algorithms give multiple models (different random seed)
- **Synthetic Tests**
  - Invert synthetic data -- heterogeneous or smooth synthetic model (test ability to recover features or avoid artifacts), add noise

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**
  - Vary amount of smoothing or damping, add in null vectors to generate additional models, change model parameterization
  - Present end-member models (e.g. smoothest model, minimum-moment model, GI solution)
  - Many nonlinear algorithms give multiple models (different random seed)
- **Synthetic Tests**
  - Invert synthetic data -- heterogeneous or smooth synthetic model (test ability to recover features or avoid artifacts), add noise
  - Test if inversion can fit data without certain features

Strategies for Uncertainty Assessment

- **Jackknife / Bootstrap** (invert subset of data -- may not work well for underdetermined problems)
- **Compute Resolution and Covariance Matrices**
- **Present Multiple Models**
  - Vary amount of smoothing or damping, add in null vectors to generate additional models, change model parameterization
  - Present end-member models (e.g. smoothest model, minimum-moment model, GI solution)
  - Many nonlinear algorithms give multiple models (different random seed)
- **Synthetic Tests**
  - Invert synthetic data -- heterogeneous or smooth synthetic model (test ability to recover features or avoid artifacts), add noise
  - Test if inversion can fit data without certain features
  - Participate in Blind Test !