What did We Learn from the Exercise of the SPICE BlindTest I?

Guangfu Shao[1,2], Chen Ji [1,2] ,Daniel Lavallee[2]

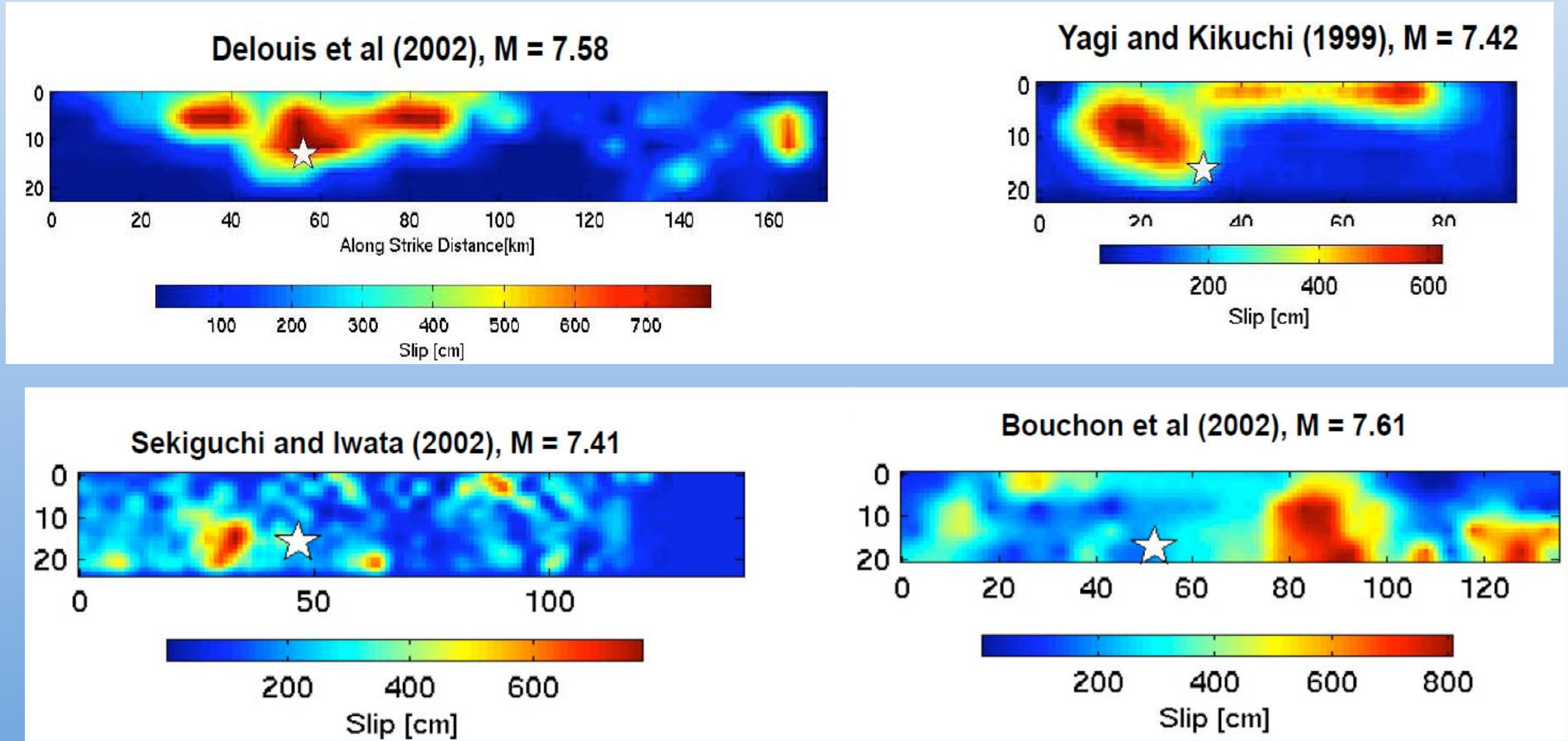
1. Department of Earth Science, University of California, Santa Barbara
2. Institute for Crustal Study, University of California, Santa Barbara

Acknowledgment: P. M. Mai

Motivation

For some earthquakes, source models obtained by different research groups do not agree with each other.

Four models for the 1999 Mw 7.5 Izmit Earthquake



(Mai et al, 2007)

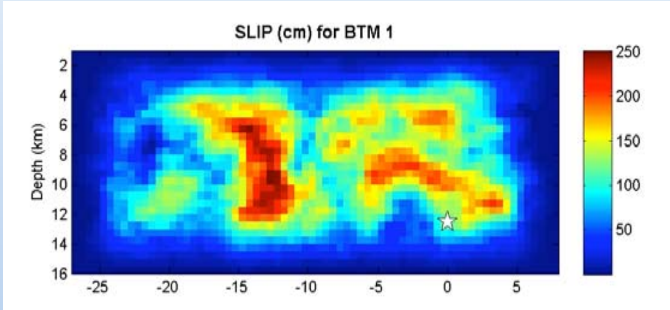
SIV BlindTest I (Mai et al., 2007)

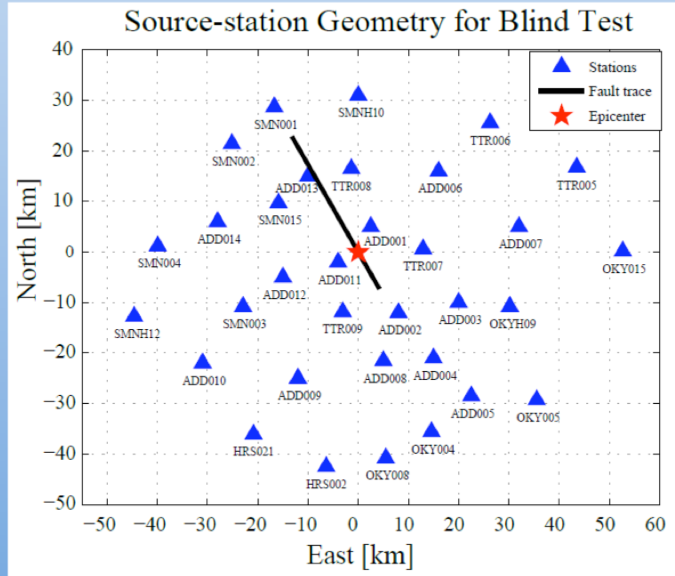
- **Data**
  1: Seismic data in velocity (*fmax* ~ 3 Hz)
  2: Static displacements

- **Available information**
  1: Fault geometry & Hypocentral location
  (strike, dip, rake: 150°, 90°, 0°)
  2: Total seismic moment:
  1.43×10^{26} dyne-cm
  3: Velocity structure
  4: Rupture does not break the surface

- **To be resolved:**
  1. Slip distribution on the fault plane
  2. Rupture velocity & rise time (both are constant; the investigators were given this information but not the values)

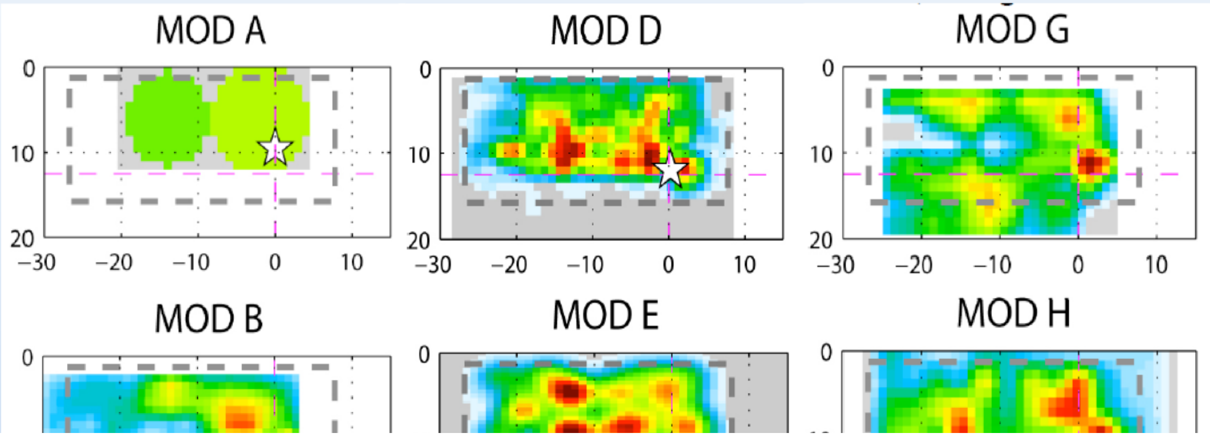
Input model





BlindTest website: http://www.seismo.ethz.ch/staff/martin/BlindTest.html

SIV BlindTest I: previous results (Mai et al., 2007)



Motivation of this study:

What are the causes of the differences

& how can they be improved?

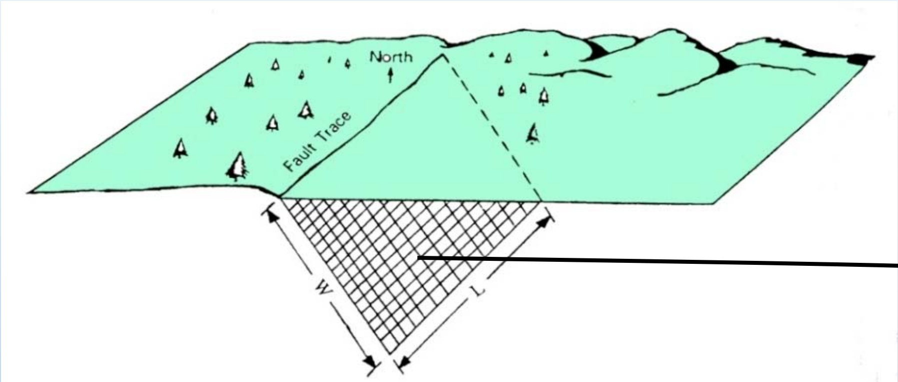
"4 out of 9 inversion results are, statistically speaking, not better than a random model with somehow correlated slip" (Mai et al., 2007)

Two questions to be addressed

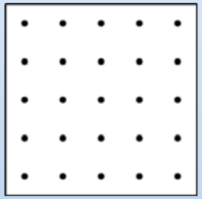
1. The quality of the source inversions depends on the number of independent constraints used during the inversions.
   Does the scheme of waveform inversions we used take fully advantage of the independent information embedded in the waveform data?

2. Usually only band-passed strong motion data are used during the finite fault source study.
   Could fitting the data in some frequency range define the source spectrum at other frequencies?

Source representation (Ji et al., 2002, 2003)



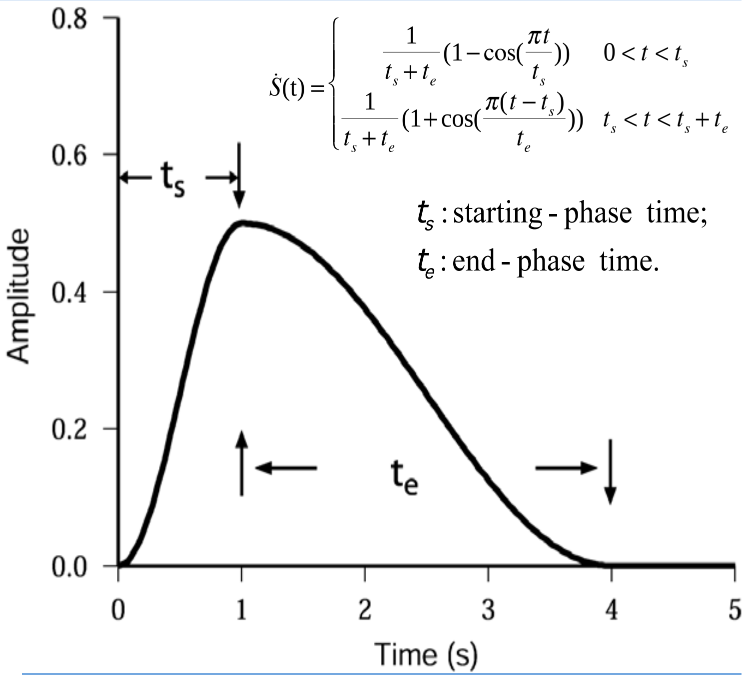
$$Y^i_{jk}(t,t',\vec{x}) = \sum_p G^i_{jk}(\vec{x}'_p,\vec{x},t) * \delta(t-\Delta t^p_{jk}-t')$$



$$u(t,\vec{x}) = \int_{-\infty}^{\infty} d\tau \iint_{\Sigma} [u_i(\xi,\tau)] c_{ijpq} \partial G_{np}(\vec{x},t-\tau;\xi,0)/\partial\xi_q \, d\Sigma$$

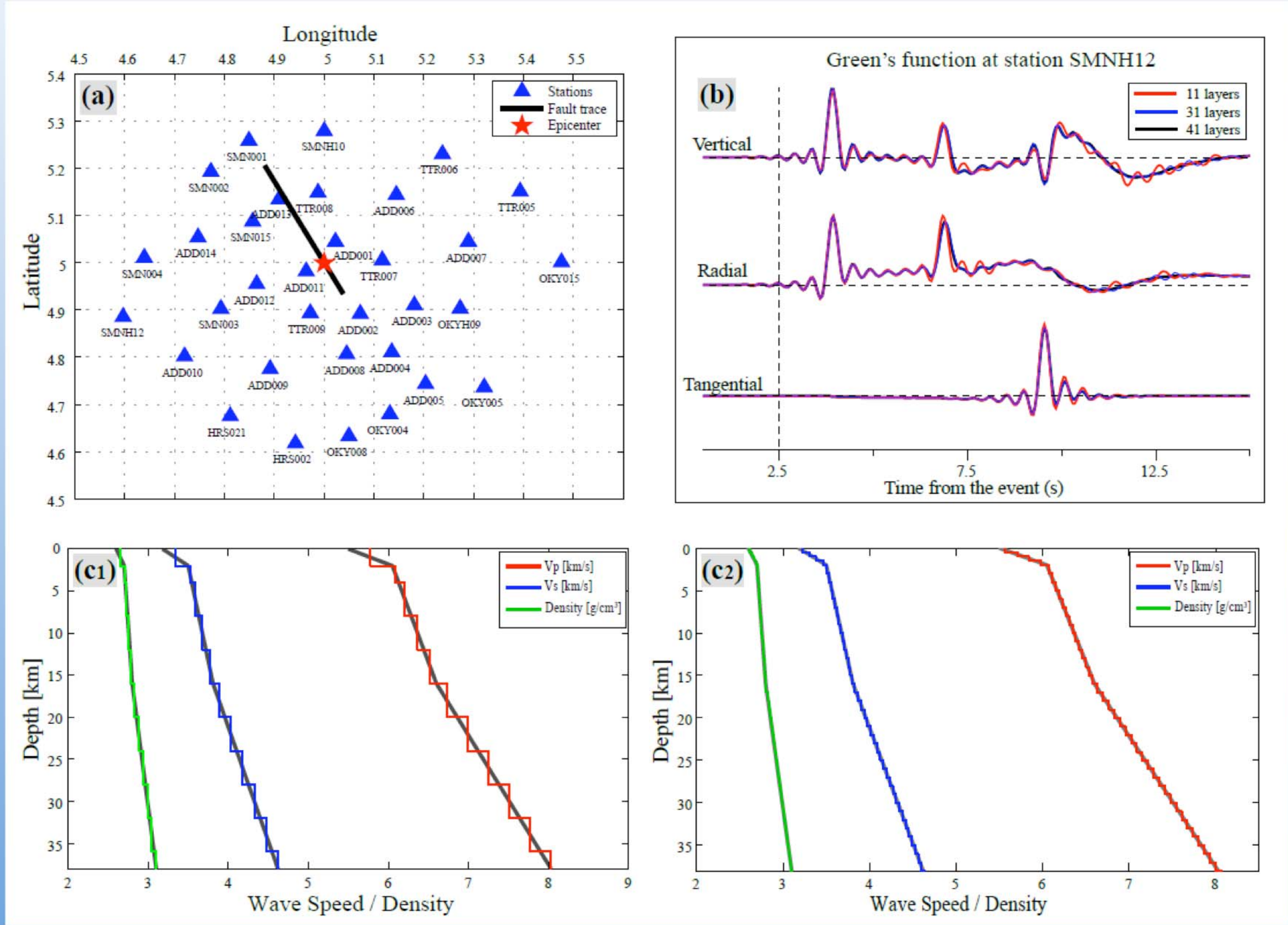
$$u(t,\vec{x}) \approx \sum_{j=1}^{n}\sum_{k=1}^{m} D_{jk}[\cos(\lambda_{jk})Y^1_{jk}(t,t',\vec{x}) + \sin(\lambda_{jk})Y^2_{jk}(t,t',x)] * \dot{S}_{jk}(t)$$

D_{jk}	Slip amplitude
λ_{jk}	Rake angle
$\dot{S}_{jk}(t)$	Derivative rise time function
t'	Rupture initiation time
$Y^i_{jk}(t,t',\vec{x})$	Subfault Green's functions



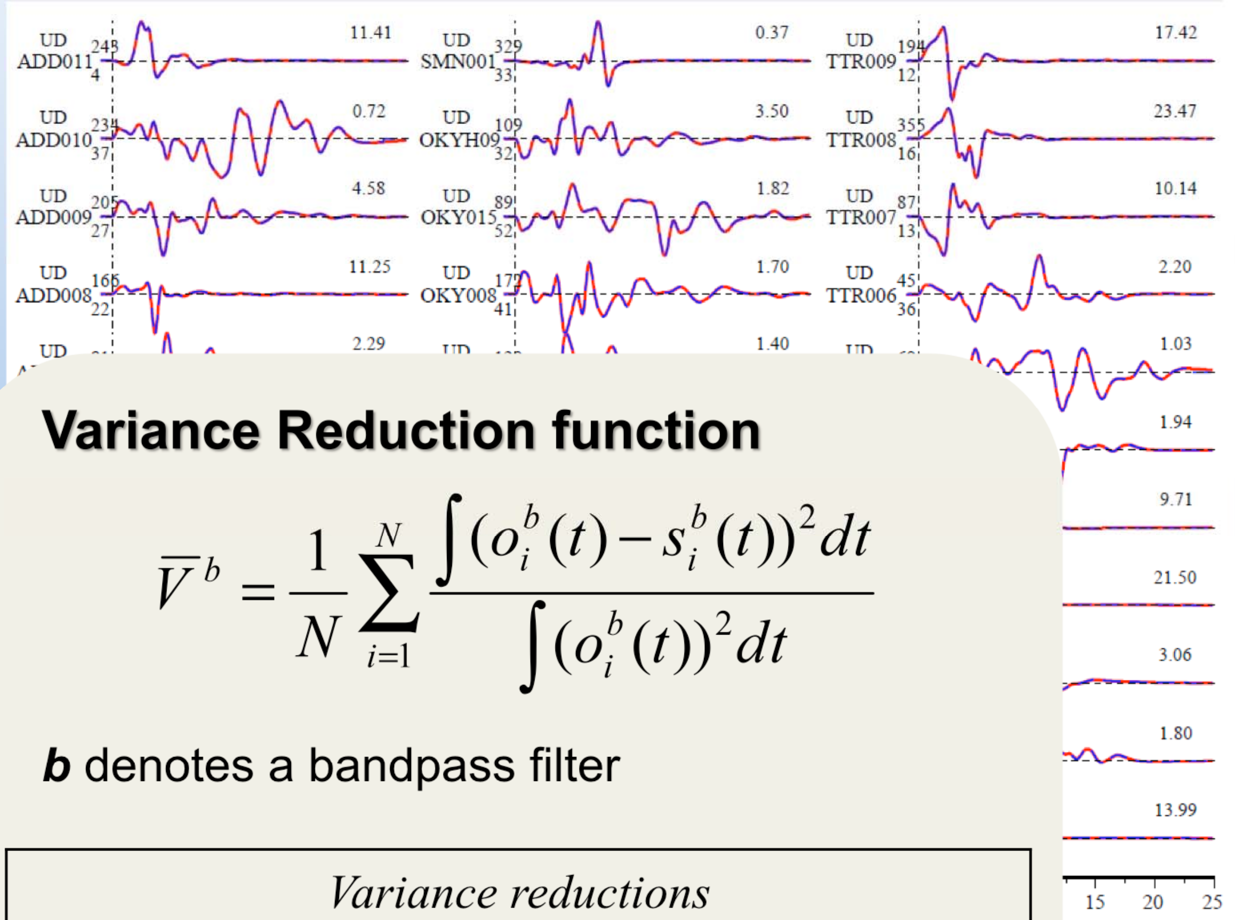
Rise time function *S(t)*

Quality control: Green's function



Forward Calculation

Vertical components in velocity



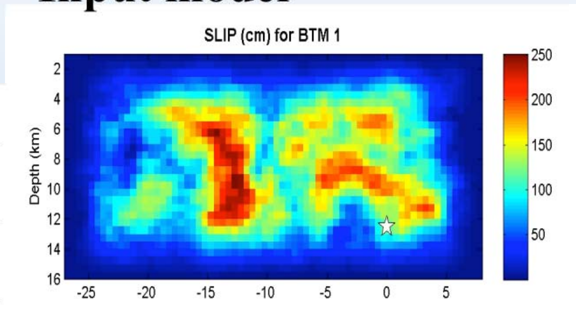
Variance Reduction function

$$\overline{V}^{b} = \frac{1}{N}\sum_{i=1}^{N}\frac{\int (o_i^b(t) - s_i^b(t))^2 dt}{\int (o_i^b(t))^2 dt}$$

b denotes a bandpass filter

Variance reductions			
0-2.0 (Hz)	*0-0.1 (Hz)*	*0.1-1.0 (Hz)*	*1.0-2.0 (Hz)*
99.91%	**99.98%**	**99.92%**	**97.53%**

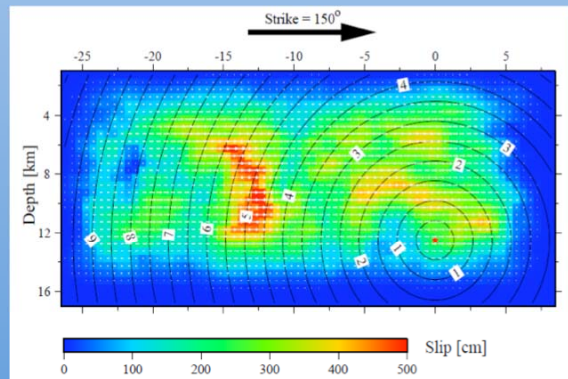
Input model



Corrected data

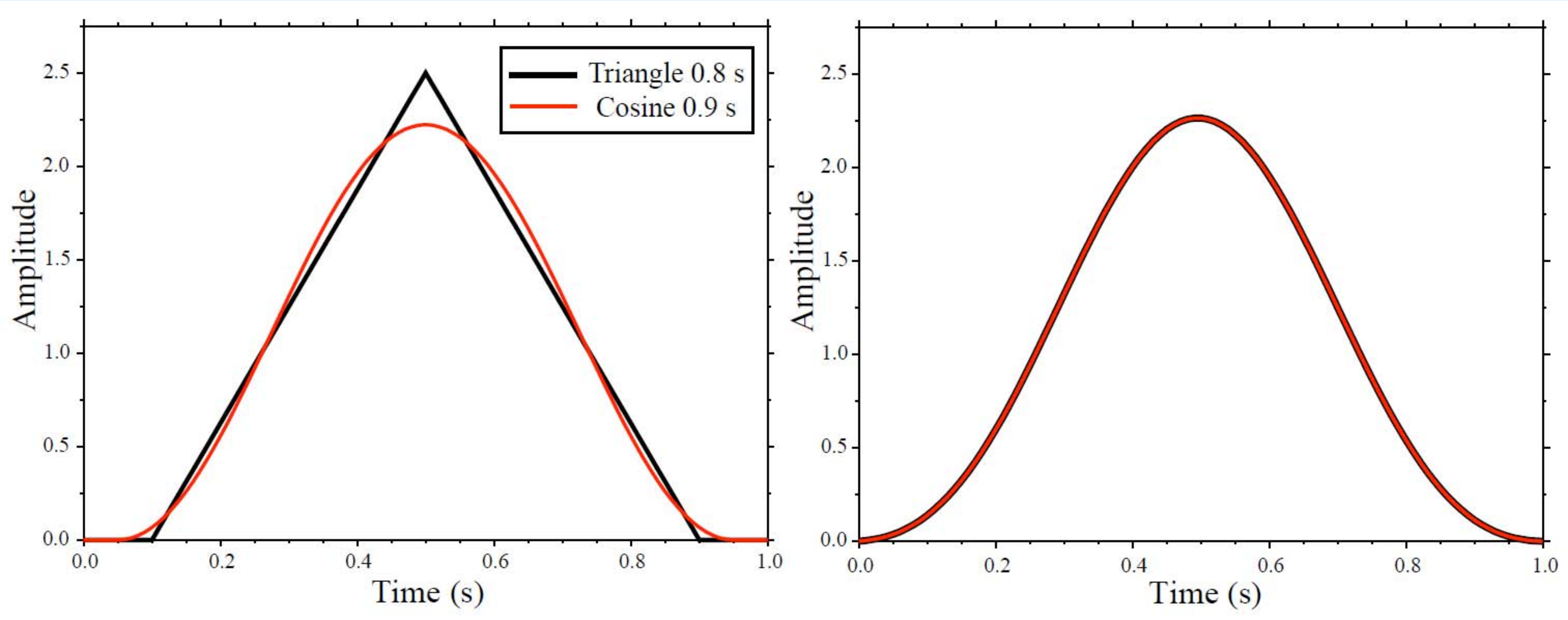
Double of our synthetics

Target



Rise time function

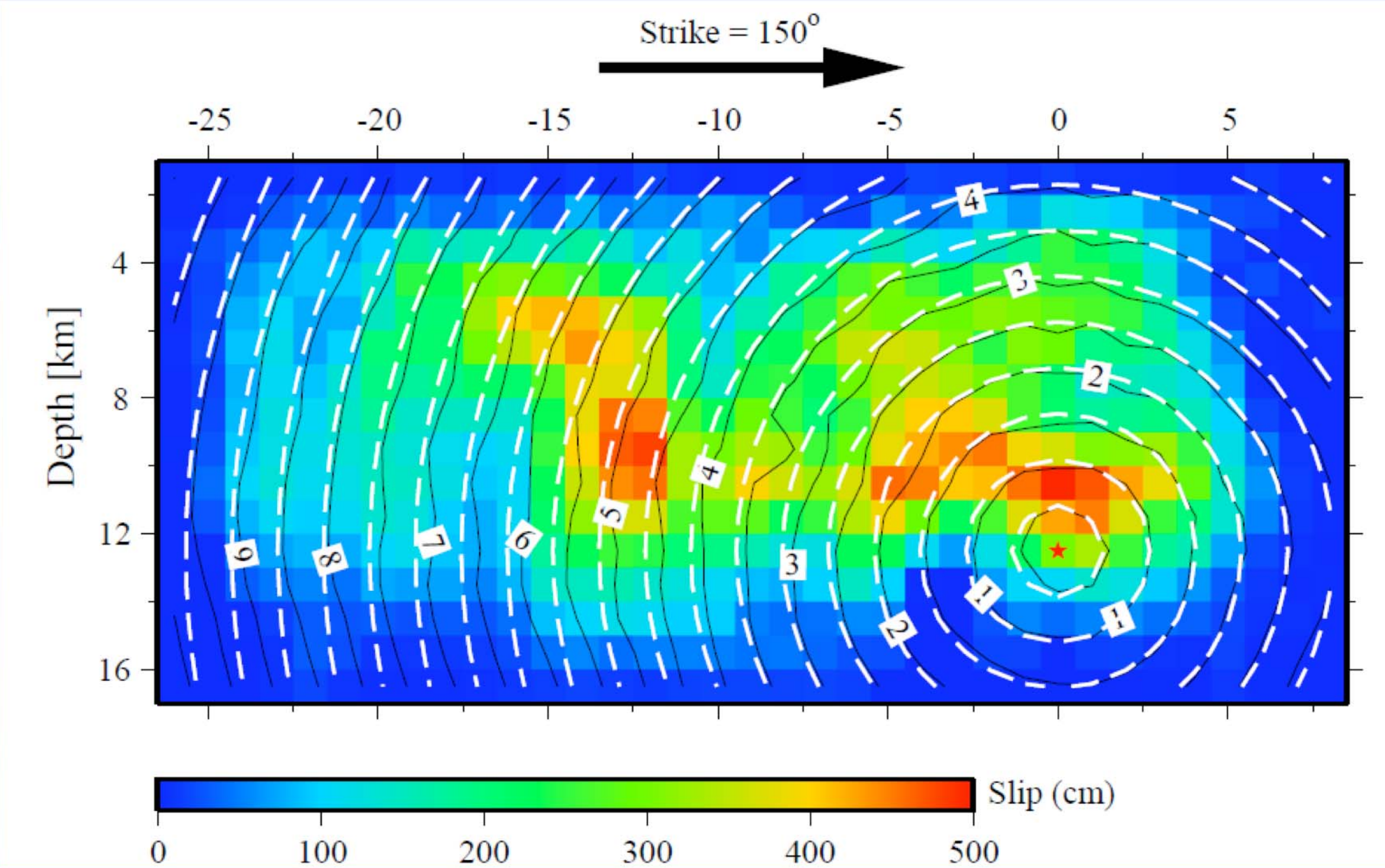
a 0.8-s triangle function VS a 0.9-s cosine function



Frequency band	0-50Hz	0-2Hz
Variance reduction:	99.659%	99.996%

Rupture velocity: Model 0

rupture velocity range : 2.2 -3.1 km/s



Inverted average rupture velocity: ~2.7 km/s

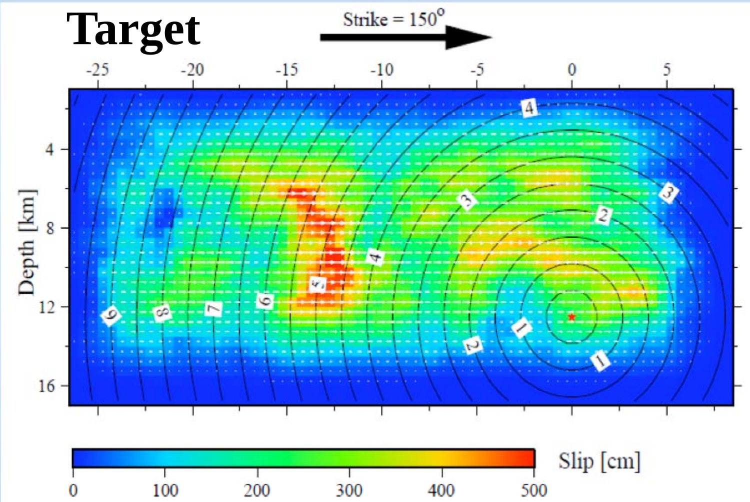
Inversion setup ➔ model space (*M*)

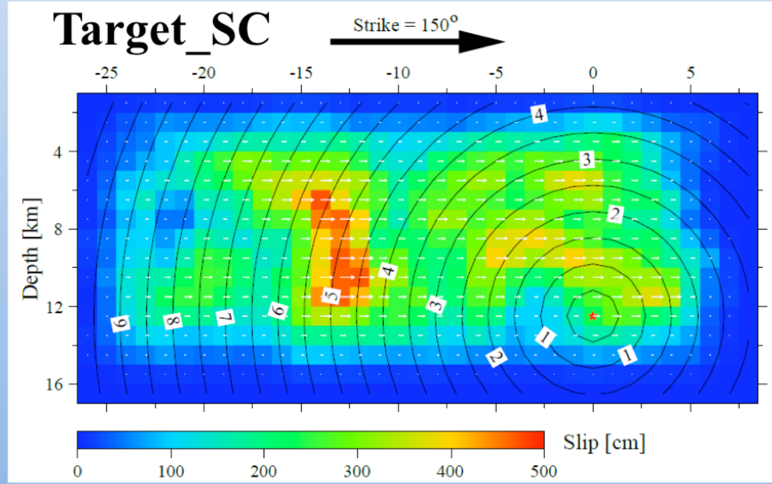
Origin Target model:

Grid size: 0.5km by 0.5km
Rise time: 0.8 sec
(symmetric triangle)
Rupture velocity: constant

Inverted Models:

Subfault size : 1km by 1km
Rise time: starting time: 0.1 s -0.8 s
ending time: 0.1 s -0.8 s
Rupture velocity: 2.65 – 2.75 km/s

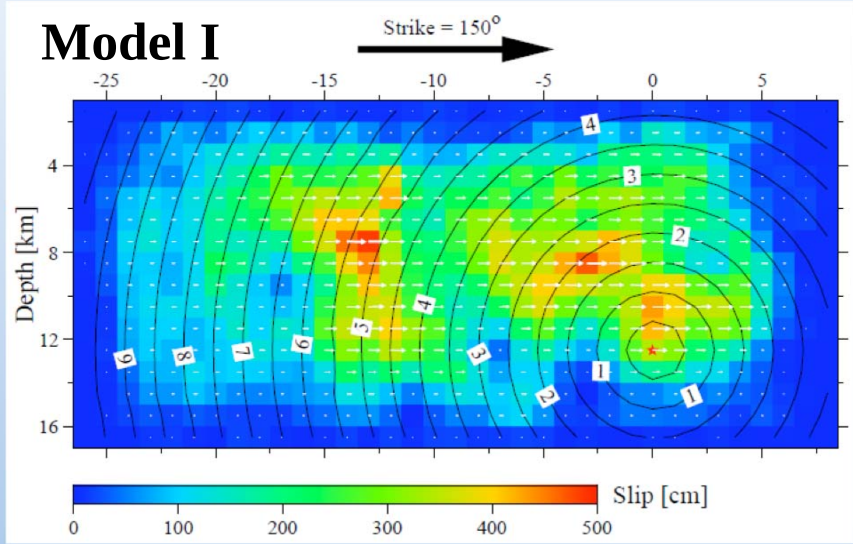


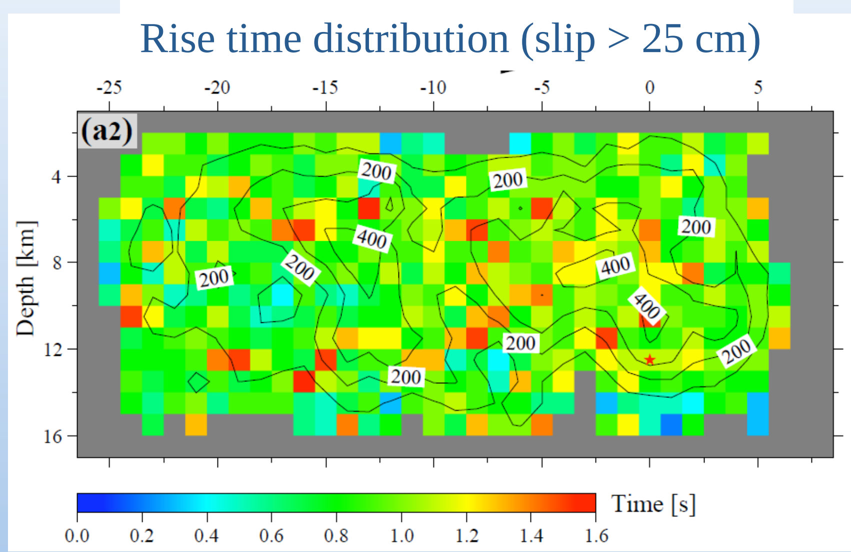


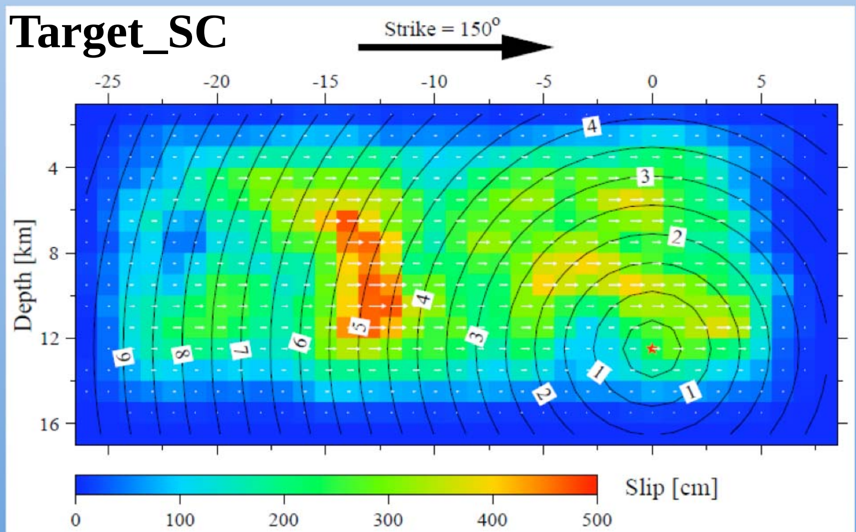
Inside the model space ***M*,**

Target_SC is the model which best approximates the **Target** model, but is it also the model which matches the data best?

Model I







	Model I	Target_SC
Peak slip	4.8 m	4.7 m
Total moment	2.72×10^{26} dyne.cm	2.86×10^{26} dyne.cm
Rise time	0.92±0.2 s	0.9 s

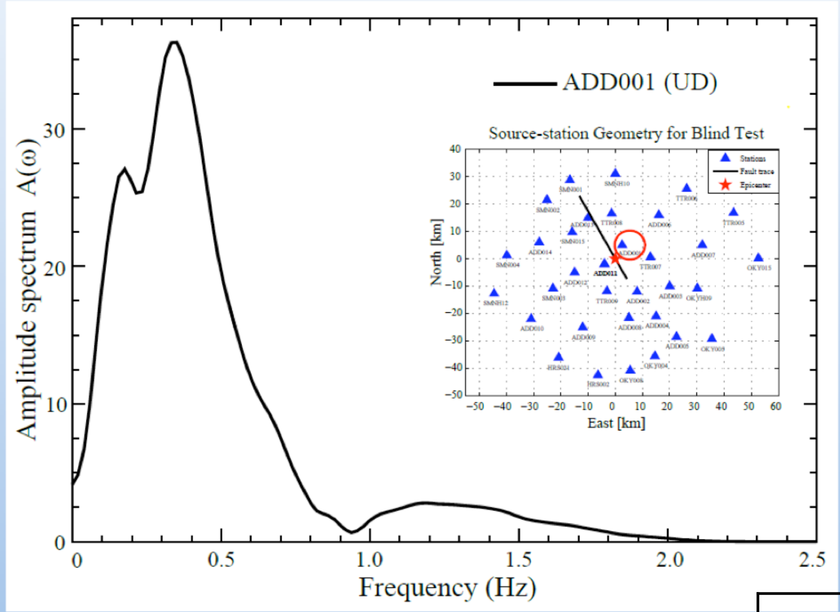
Can we further improve the result?

Models	*Variance reductions*			
	0-2.0 (Hz)	*0-0.1 (Hz)*	*0.1-1.0 (Hz)*	*1.0-2.0 (Hz)*
Target	99.91%	99.98%	99.92%	97.53%
Target_SC	99.32%	99.72%	99.45%	86.21%
Model I	99.35%	99.28%	99.61%	77.02%

Statement I

Inside the model space ***M*** defined by our source representation, **Target_SC** is the model which is closest to the **Target** model, but it is **NOT** the model which fits the data best in **a term of variance reduction**.

Spectrum: Energy Ratio



Average relative energy

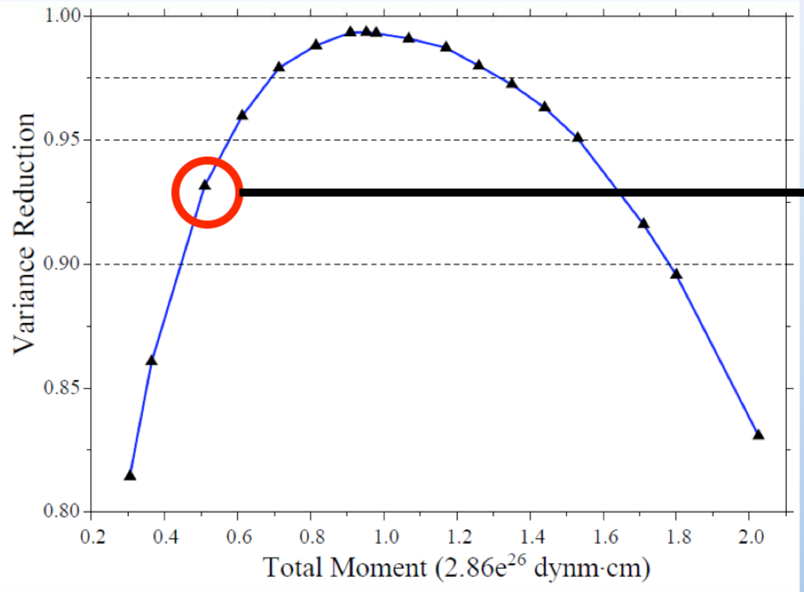
$$\overline{R}^{b} = \frac{100}{N}\sum_{i=1}^{N}\frac{\int (o_i^b(t))^2 dt}{\int (o_i(t))^2 dt}$$

Relative Energy			
0-2.0 (Hz)	*0-0.1 (Hz)*	*0.1-1.0 (Hz)*	*1.0-2.0 (Hz)*
100%	15.04%	86.02%	2.73%

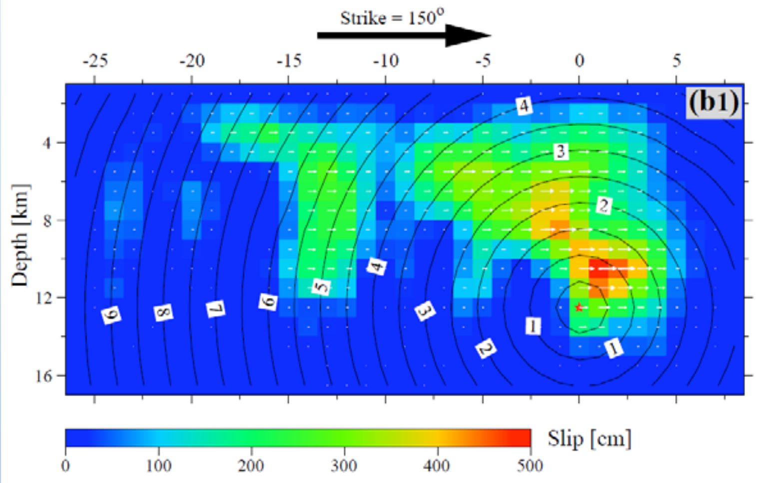
Note:

Misfit functions, such as variance reduction, are designed to catch the difference in amplitude (or energy). Therefore, for this case, it is dominated by the signals from 0.1 to 1 Hz.

Sensitivity to the total seismic moment



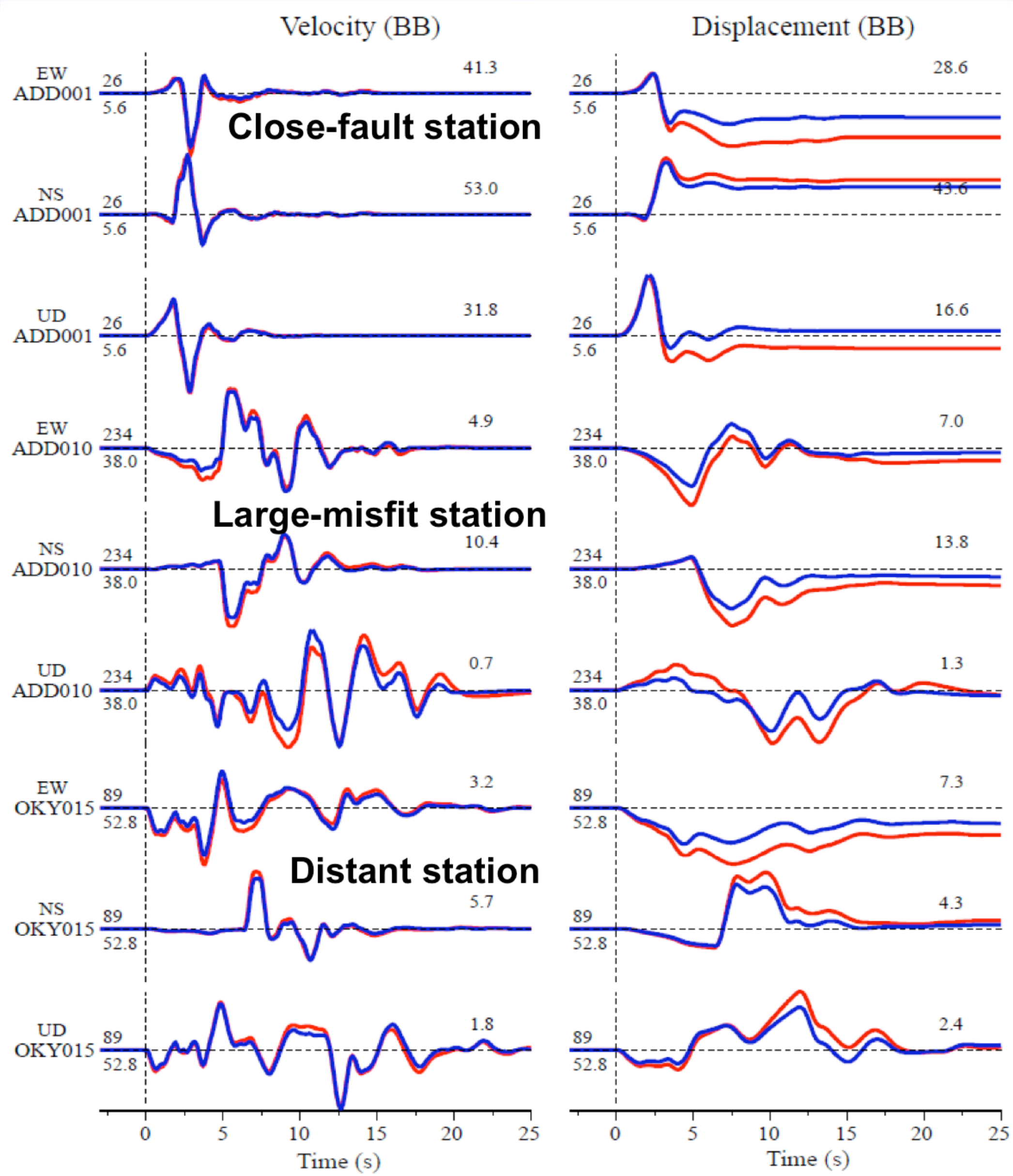
Model II: Mo=1.46×10^{26} dyne.cm

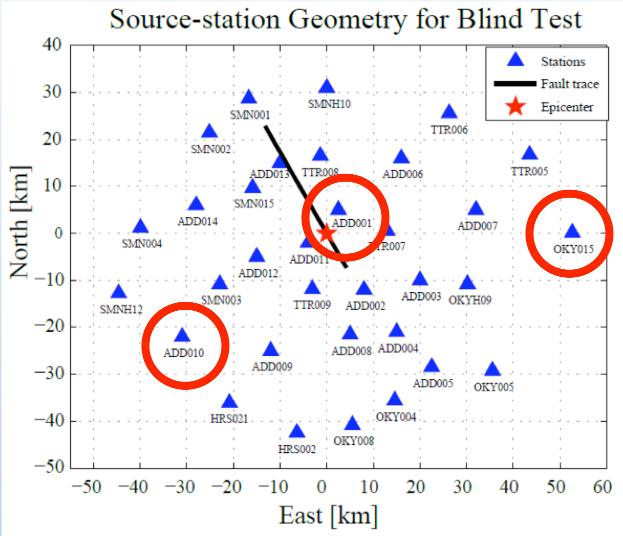


Models	*Variance reductions*			
	0-2.0 (Hz)	*0-0.1 (Hz)*	*0.1-1.0 (Hz)*	*1.0-2.0 (Hz)*
Target_SC	99.32%	99.72%	99.45%	86.21%
Model I	99.35%	99.28%	99.61%	77.02%
Model II	93.15%	76.86%	**95.15%**	86.36%

Model II cannot well explain the data from 0 to 0.1 Hz.

Waveform comparison

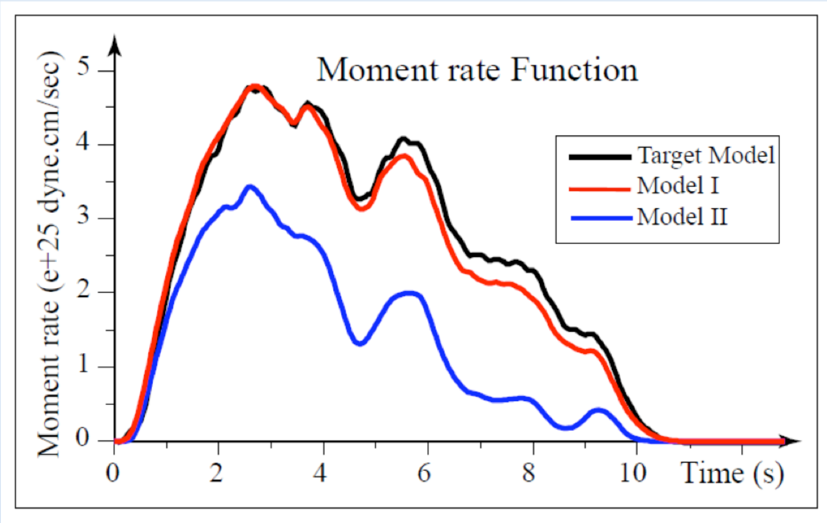


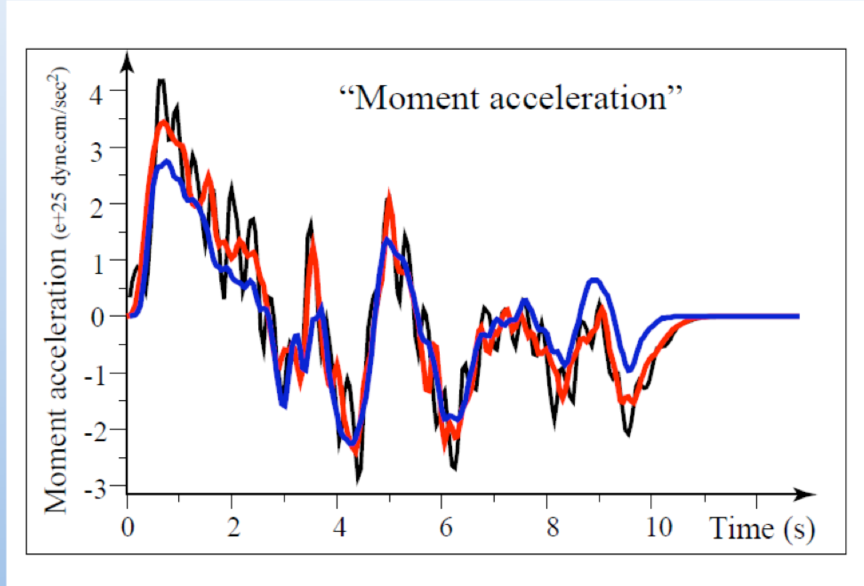


Model I

Model II: 0.5 * Mo

Comparison of moment rate functions



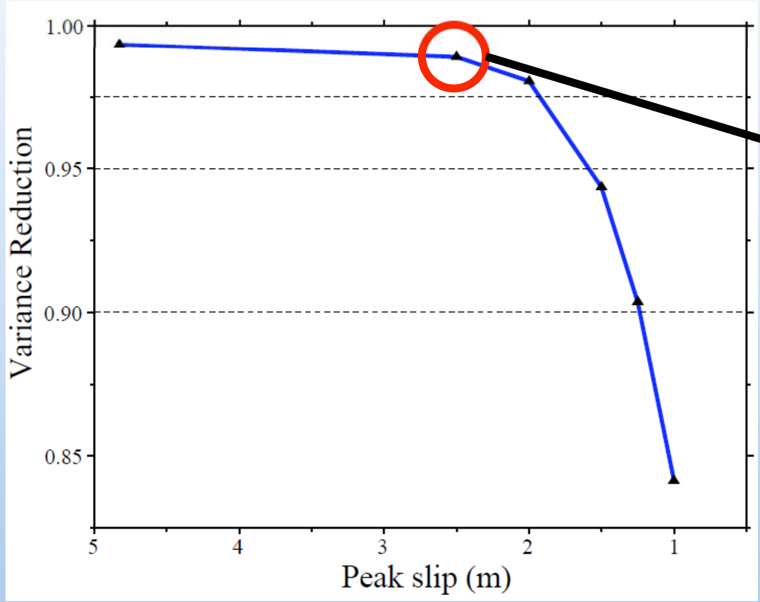


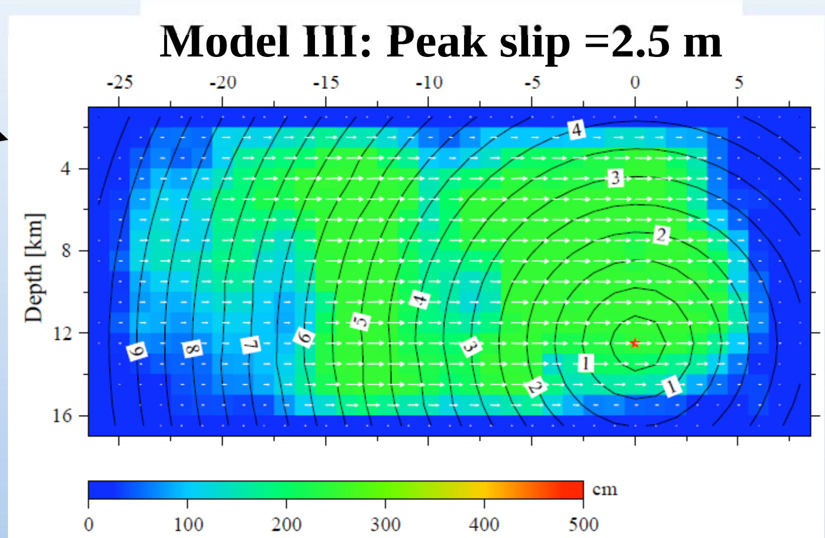
Far field body-wave

1: Displacement $U(\vec{r},t) \approx \frac{1}{4\pi\rho v^3}\psi(\theta,\phi)\frac{1}{r}\dot{M}(t)$

2: Velocity $V(\vec{r},t) \approx \frac{1}{4\pi\rho v^3}\psi(\theta,\phi)\frac{1}{r}\ddot{M}(t)$

Sensitivity to the peak slip





Models	Variance reductions			
	0-2.0 (Hz)	*0-0.1 (Hz)*	*0.1-1.0 (Hz)*	*1.0-2.0 (Hz)*
Target_SC	99.32%	99.72%	99.45%	86.21%
Model I	99.35%	99.28%	99.61%	77.02%
Model III	98.87%	98.90%	99.36%	64.26%

Model III cannot match the signals from 1 to 2 Hz.

Some intuitive thoughts

In the frequency domain, the distribution of the independent constraints for source inversions that we could obtain from seismic waveforms is not uniform.

✓For a single broadband waveform, let us assume that the amount of constraints embedded in **SP** (1.0 Hz to 2 Hz) band is compatible with that of the **DM** band (0.1 Hz to 1 Hz), because of their similar bandwidths.

✓Adding more stations should boost the total amount of constraints. But not all of them are independent with others.

✓Considering the fact that two close stations more likely have similar long period waveforms than short period waveforms, the increasing of independent constraints from the **SP** band should be much more significantly than from the **DM** band.

✓For a very dense strong motion seismic network, we then argue that there are much more independent constraints embedded in the **SP** band than the **DM** band.

Two Questions to be Addressed

1. The quality of the source inversions depends on the number of independent constraints used during the inversions.
   Does the scheme of waveform inversions we used take full advantage of the independent information embedded in the waveform data?

NO!!!

2. Usually only band-passed strong motion data are used during the finite fault source study.
   Could fitting the data in some frequency range define the source spectrum at other frequencies?

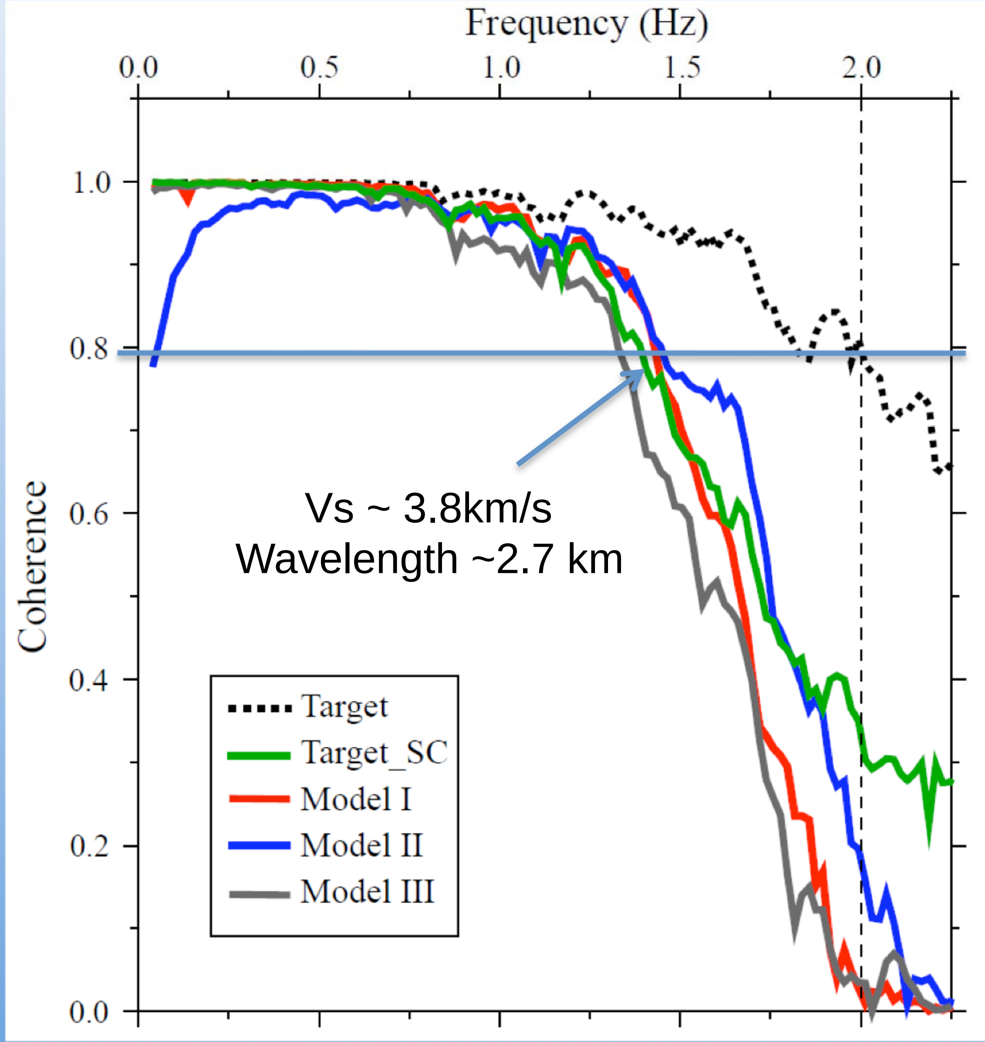
A better way to show misfits: "Coherence" function

Problem in waveform inversions:

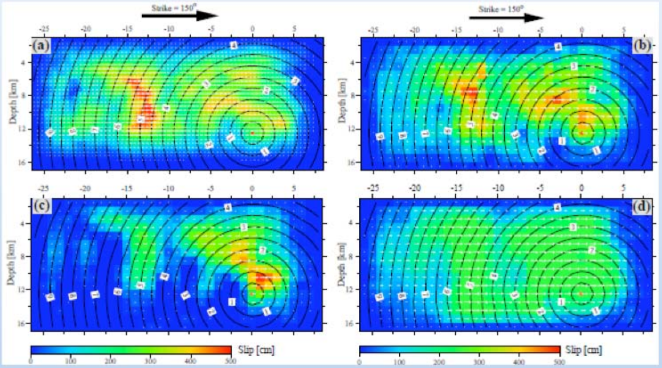
The constraints embedded in the SP band (1-2Hz) were able to be extracted ***only when*** the quality of the synthetic-data fits in SP band is acceptable. However, variance reduction function is insensitive to the misfits in high frequency.

"Coherence" function

$$e(f) = \frac{1}{N}\left|\sum_{i}^{N}\frac{2REAL[d_i(f)s_i^*(f)]}{d_i(f)d_i^*(f)+s_i(f)s_i^*(f)}\right|$$

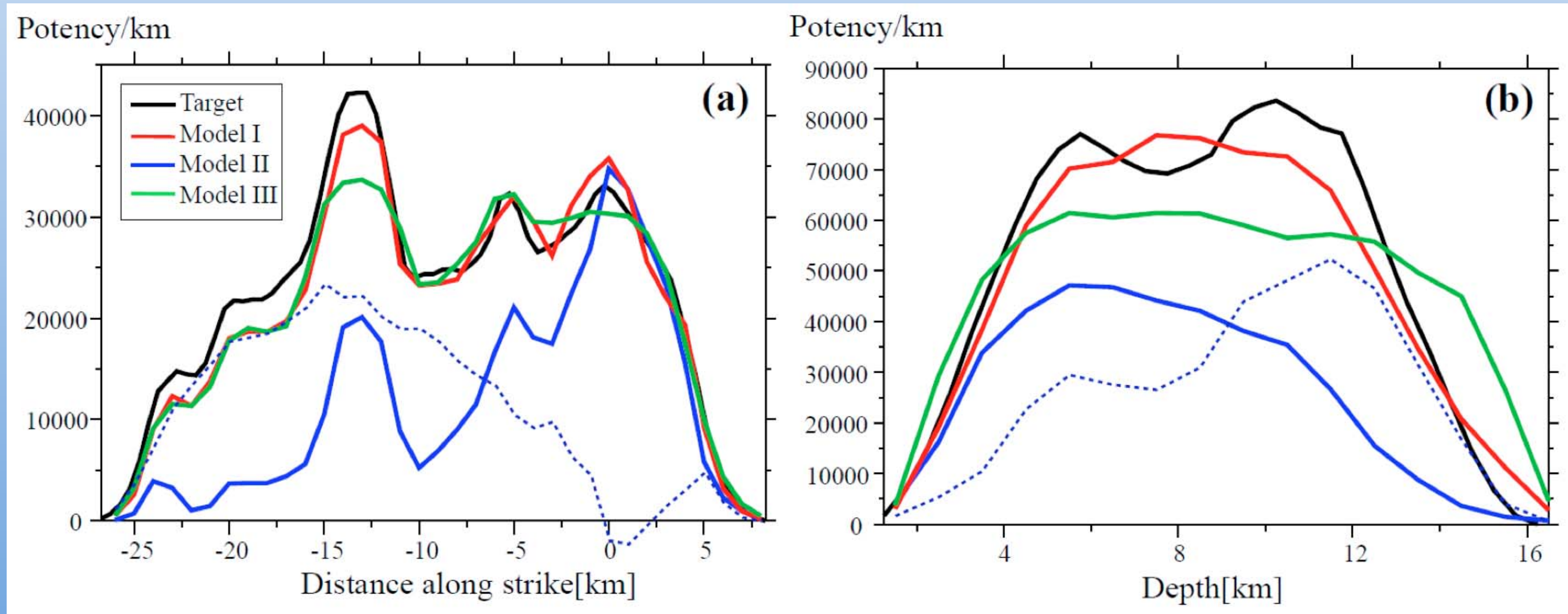


Comparison of potency density profiles



Potency = slip amplitude * slip area

Along strike spatial variation is better resolved than the resolution along the depth



Blue dot line denotes the discrepancy between the **Target** model and **Model II**.

Conclusions

We have demonstrated that the slip history of a complex strike-slip rupture on a vertical fault could be reasonably well resolved using near-fault strong motion records, but the results, particularly in more realistic circumstances, might suffer errors due to the following reasons.

1. **Simplifications of the source**. Using a large subfault could lead to error not only in the spatial distribution but also in the temporal evolution.

2. **The frequency range being inverted is as important as that of the spatial distribution of stations.** Investigators must be aware that using bandlimited seismic data can lead to erroneous results even if the synthetics have a very good fit to the data.

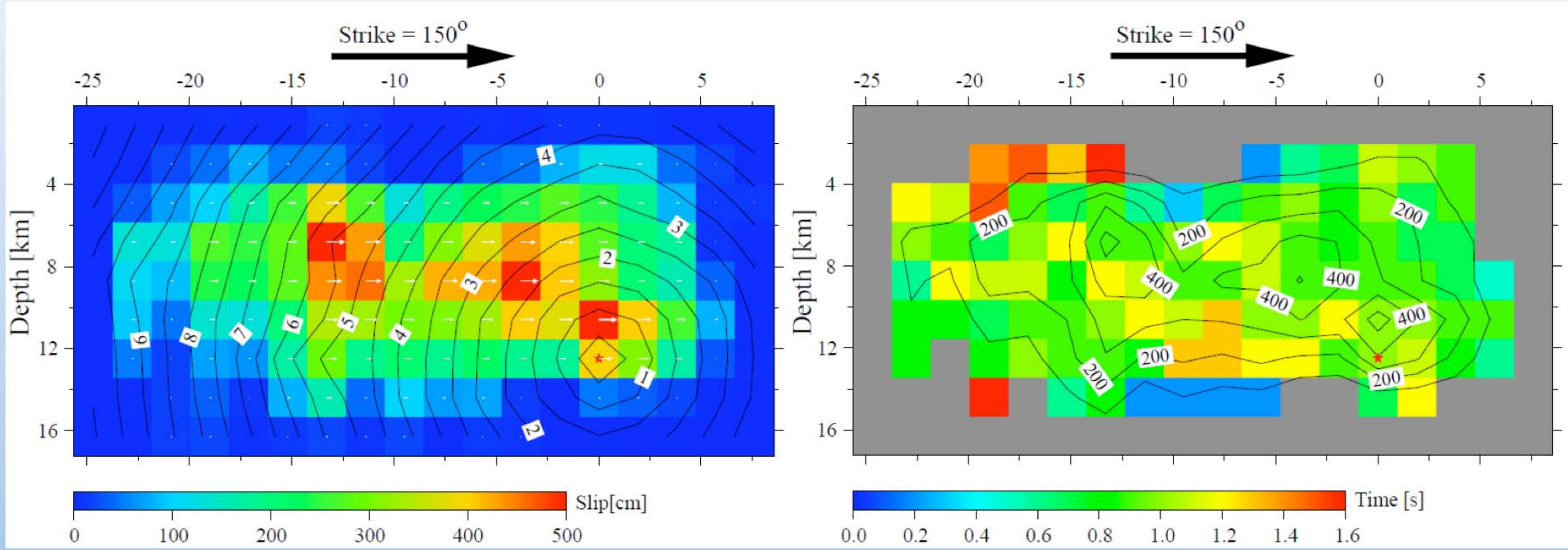
Conclusions

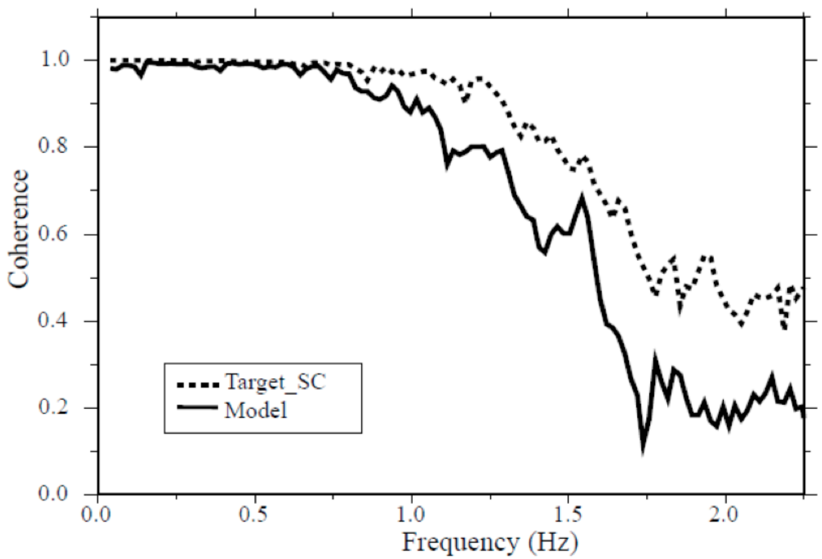
3. Errors accompany with the way that we compare the synthetics and data. The typical objective functions such as variance reduction tends to emphasize a particular frequency band of radiated seismic signals but is insensitive to the misfit within other bands. To some extent, the misfit function has a similar impact as using bandlimited data. Analogous to the earthquake location problem, the finite fault inversion tends to have better along-strike resolution than depth resolution. When we invert for particle velocity, the inversions are more sensitive to the “moment acceleration” than the “moment rate”.

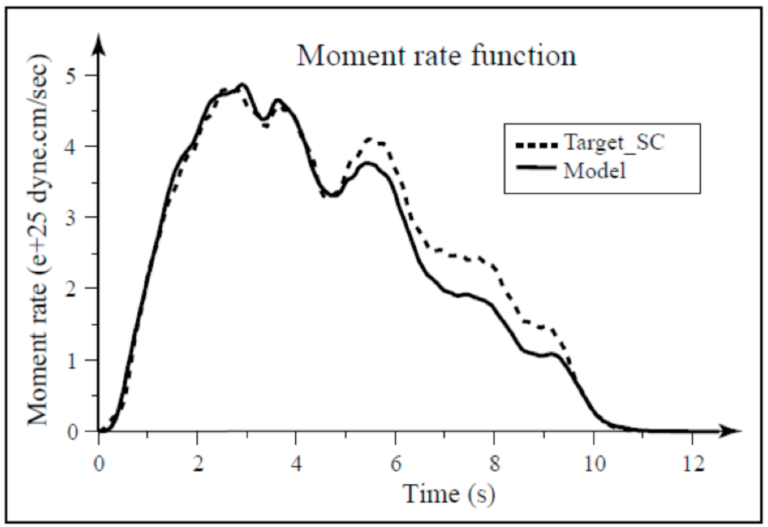
We advocate the idea of joint inversions to extend the bandwidth; and also the development of new objective functions.

Appendix

1. Inversion with a subfault size of 1.9 km by 1.9 km

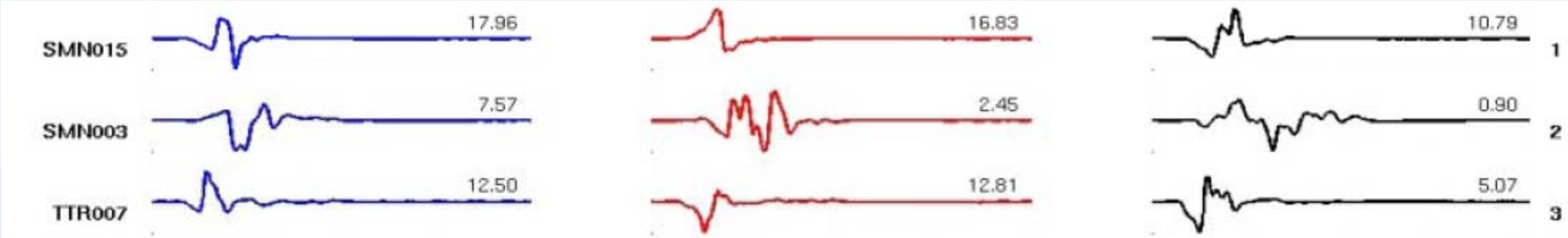






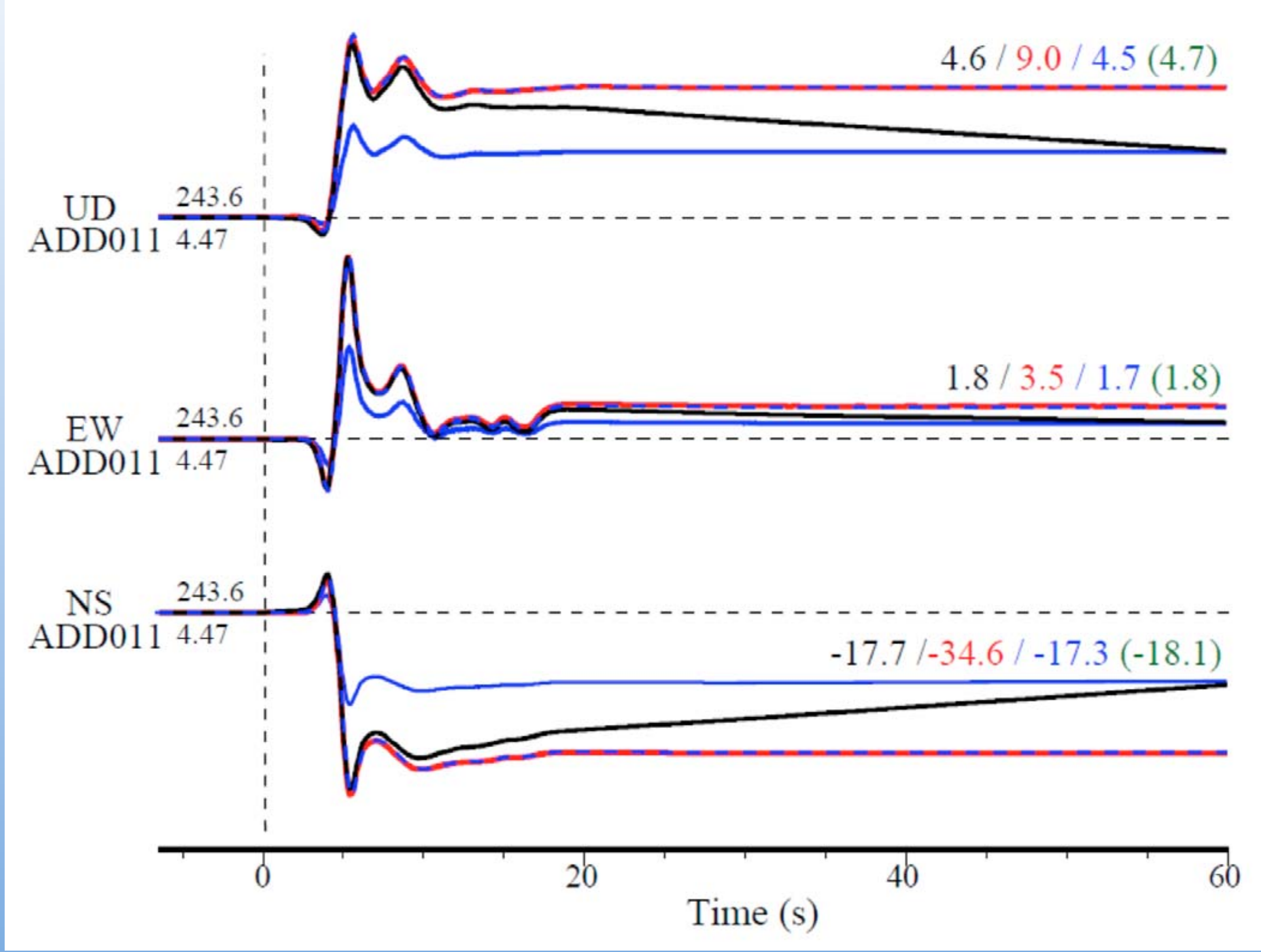
2.1 Data correction

Mai et al., 2007 AGU

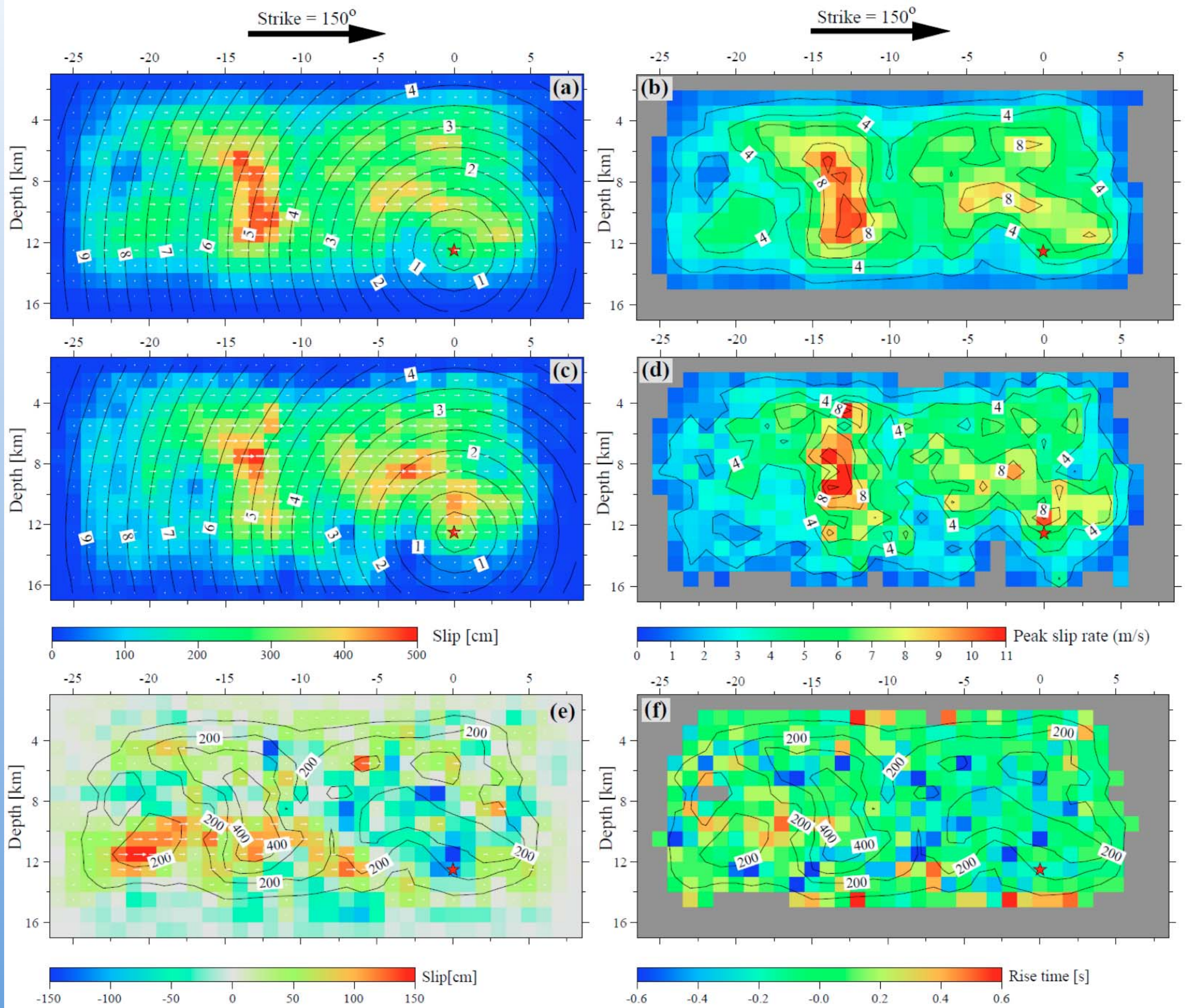


BlindTest Website

2.2 Data correction

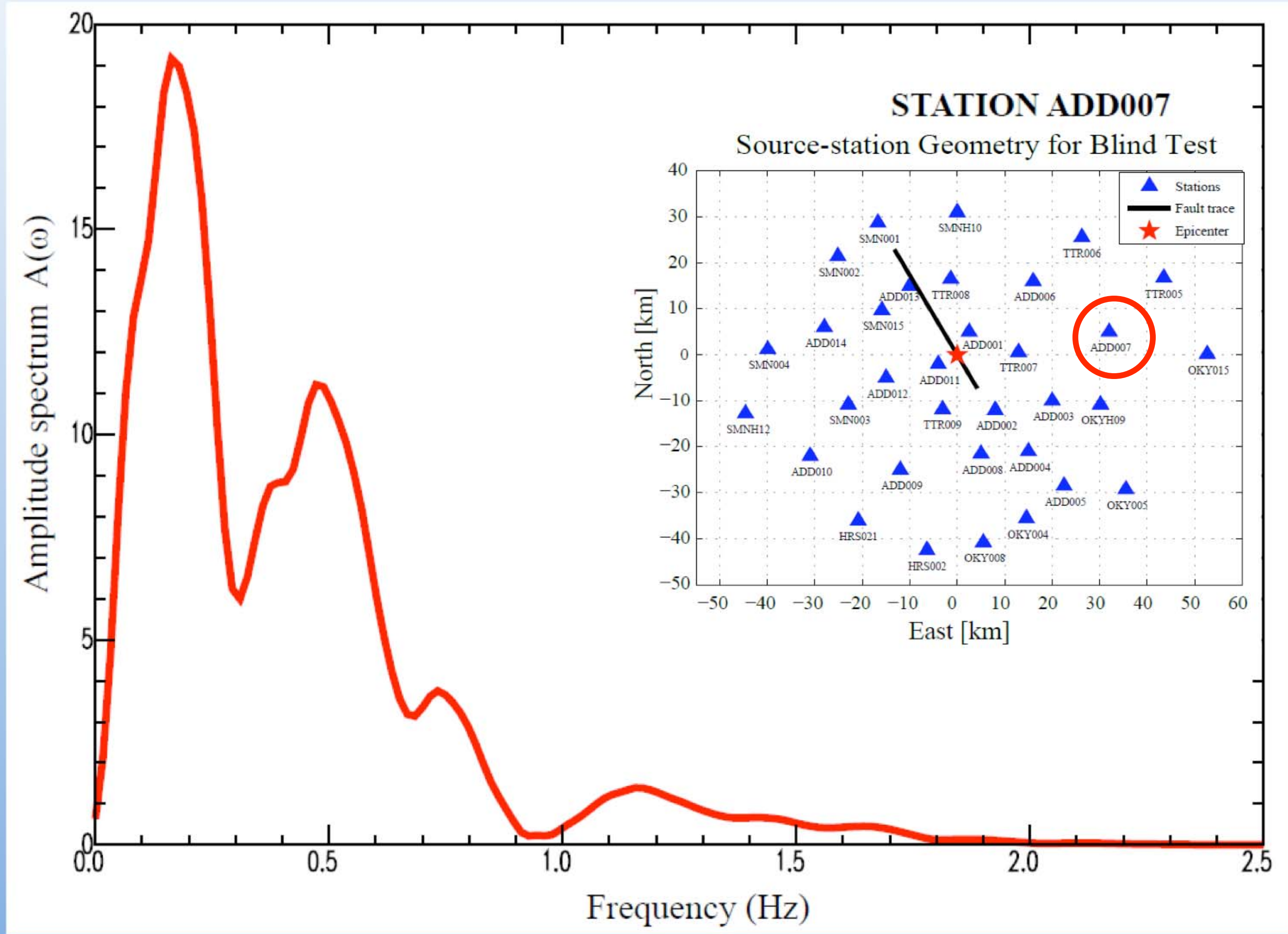


Comparison of displacement waveforms at station ADD011

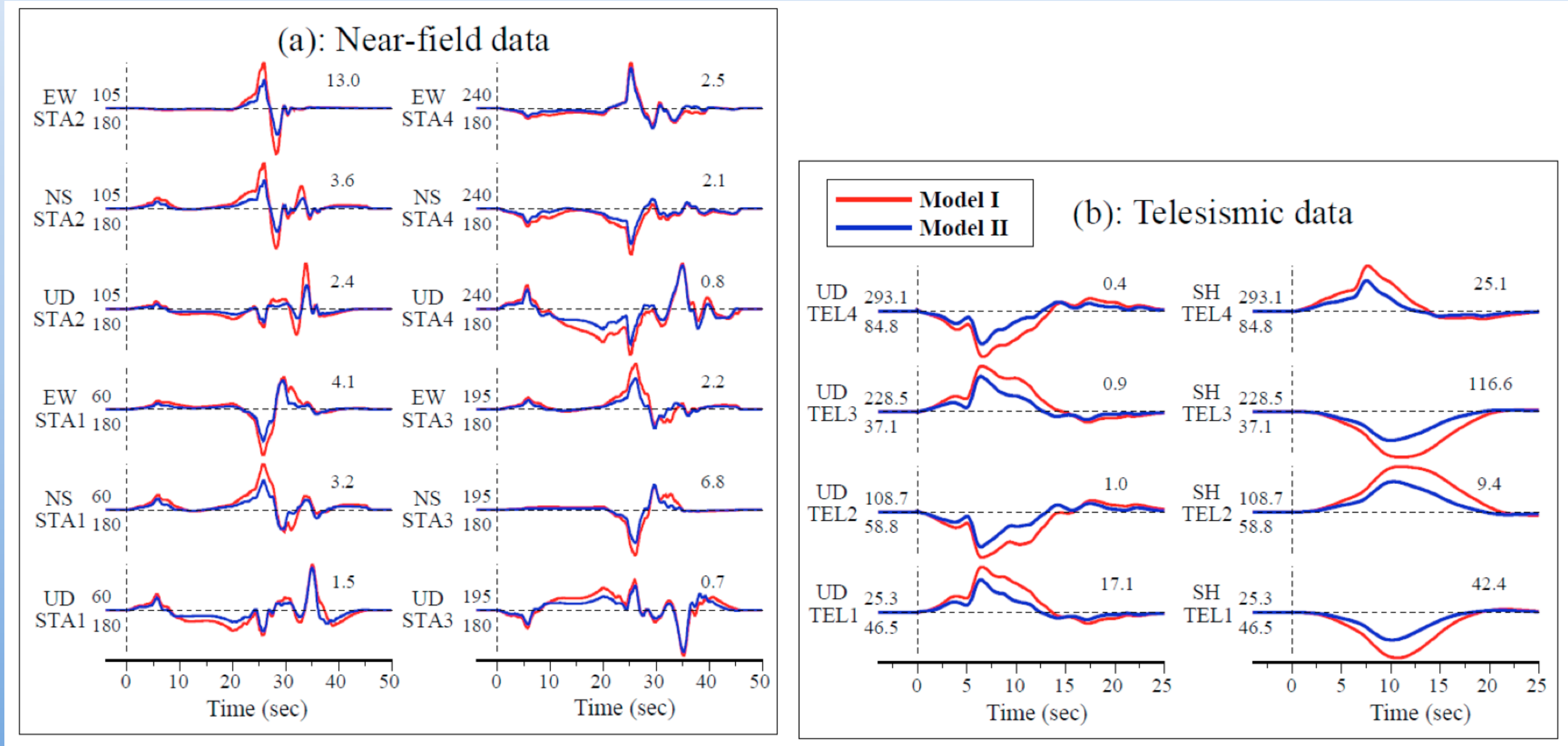


Strike = 150°
Strike = 150°
(a)
(b)
(c)
(d)
(e)
(f)
Depth [km]
Slip [cm]
Peak slip rate (m/s)
Slip[cm]
Rise time [s]

Appendix 1: Energy Ratio



The amplitude spectrum of the velocity record (NS component) at the station ADD007.



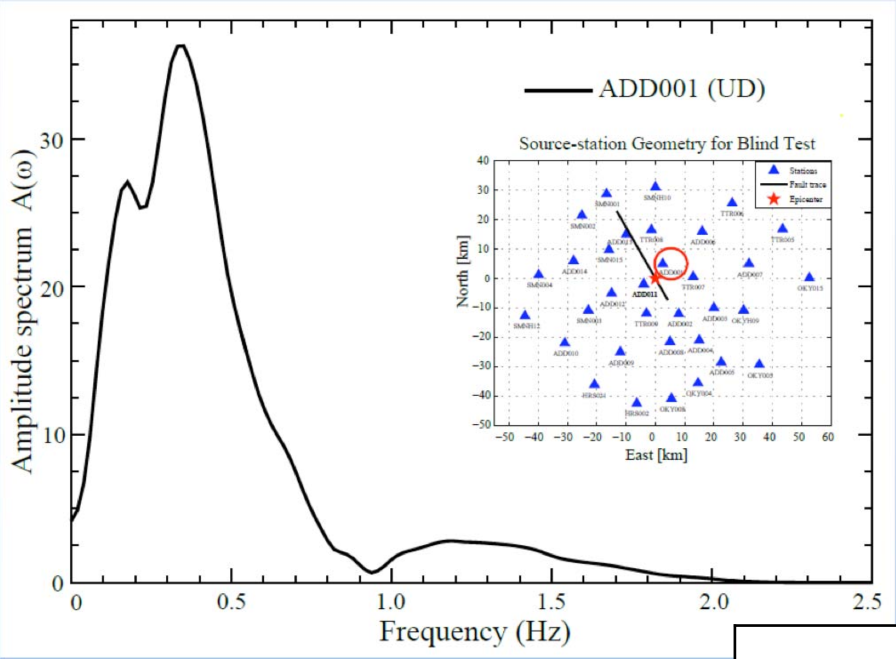
Forward predicted displacement
waveforms of **Model I** (red), **Model II** (blue)

Two Questions to be addressed

- The quality of the source inversions depends on the number of independent constraints used during the inversions. **Does the scheme of waveform inversions we used take fully advantage of the independent information embedded in the waveform data?**

- Usually only band-passed strong motion data are used during the finite fault source study. **Could fitting the data in some frequency range define the source spectrum at other frequencies?**

Energy distribution: Source Spectrum



Average relative energy

$$\overline{R}^b = \frac{100}{N}\sum_{i=1}^{N}\frac{\int (o_i^b(t))^2 dt}{\int (o_i(t))^2 dt}$$

Relative Energy			
0-2.0 (Hz)	*0-0.1 (Hz)*	*0.1-1.0 (Hz)*	*1.0-2.0 (Hz)*
100%	15.04%	86.02%	2.73%

Note:

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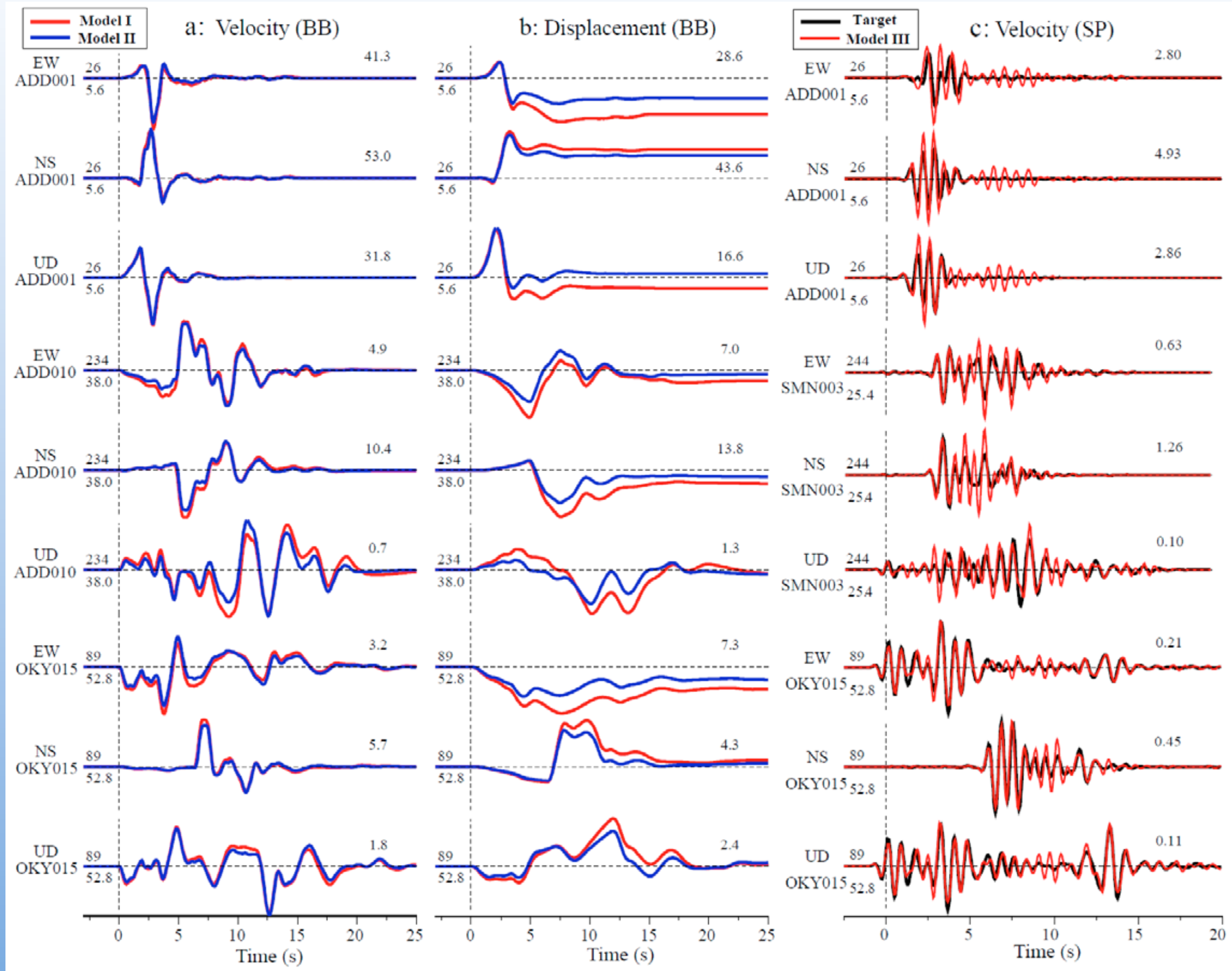

Model I
Model II
a: Velocity (BB)
b: Displacement (BB)
Target
Model III
c: Velocity (SP)
EW ADD001
NS ADD001
UD ADD001
EW ADD010
NS ADD010
UD ADD010
EW OKY015
NS OKY015
UD OKY015
EW SMN003
NS SMN003
UD SMN003
Time (s)
Time (s)
Time (s)