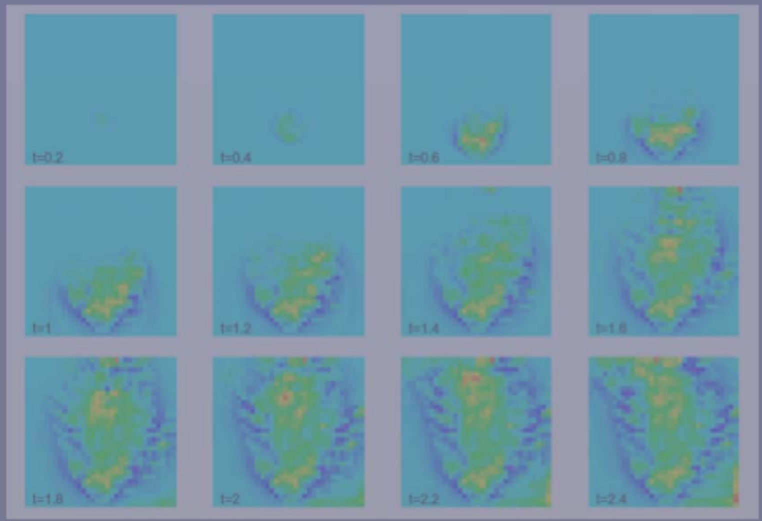
Source Inversion with Minimal Assumptions



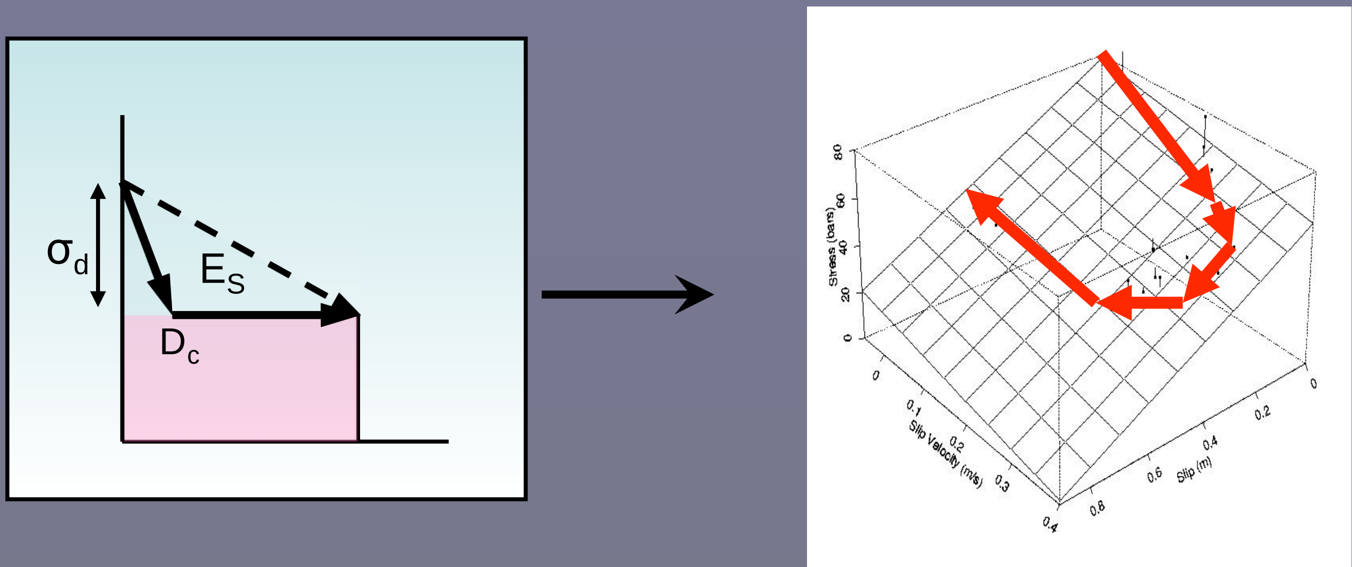
Bill Ellsworth

U. S. Geological Survey

A central goal of seismology is to understand the physical conditions under which earthquakes occur.

Key question we would like to answer include:

- Evolution of displacement and stress in space and time
- Amplitude of seismic waves at arbitrary locations
- Frictional resistance (what really happens when a fault slips?)
- Nucleation process (is anything predictable?)



The representation theorem for seismic sources defines a linear relationship between the displacement discontinuity across the fault surface, s, and the amplitude of the displacement wave field u at position x and time t.

$$u(x,t) = \int \int G(y,t-\tau)\, s(y,\tau)\, dy\, d\tau$$

The problem is discretized by replacing the fault surface with a grid of point sources for which G is the impulse response seismogram at observation point u.

$$u = G\, s$$

where u is the data vector of the seismograms, G is the design matrix containing the point source seismograms for each source-station pair and s contains the unknown time history of the displacement discontinuity for each point source. As posed, the problem is linear but underdetermined.

How do we solve this infinitely underdetermined linear system using

- Uncertain data
- Uncertain Green’s functions
- Uncertain model specification

while

- Using full bandwidth of data
- Avoiding unnecessary a-priori constraints

????

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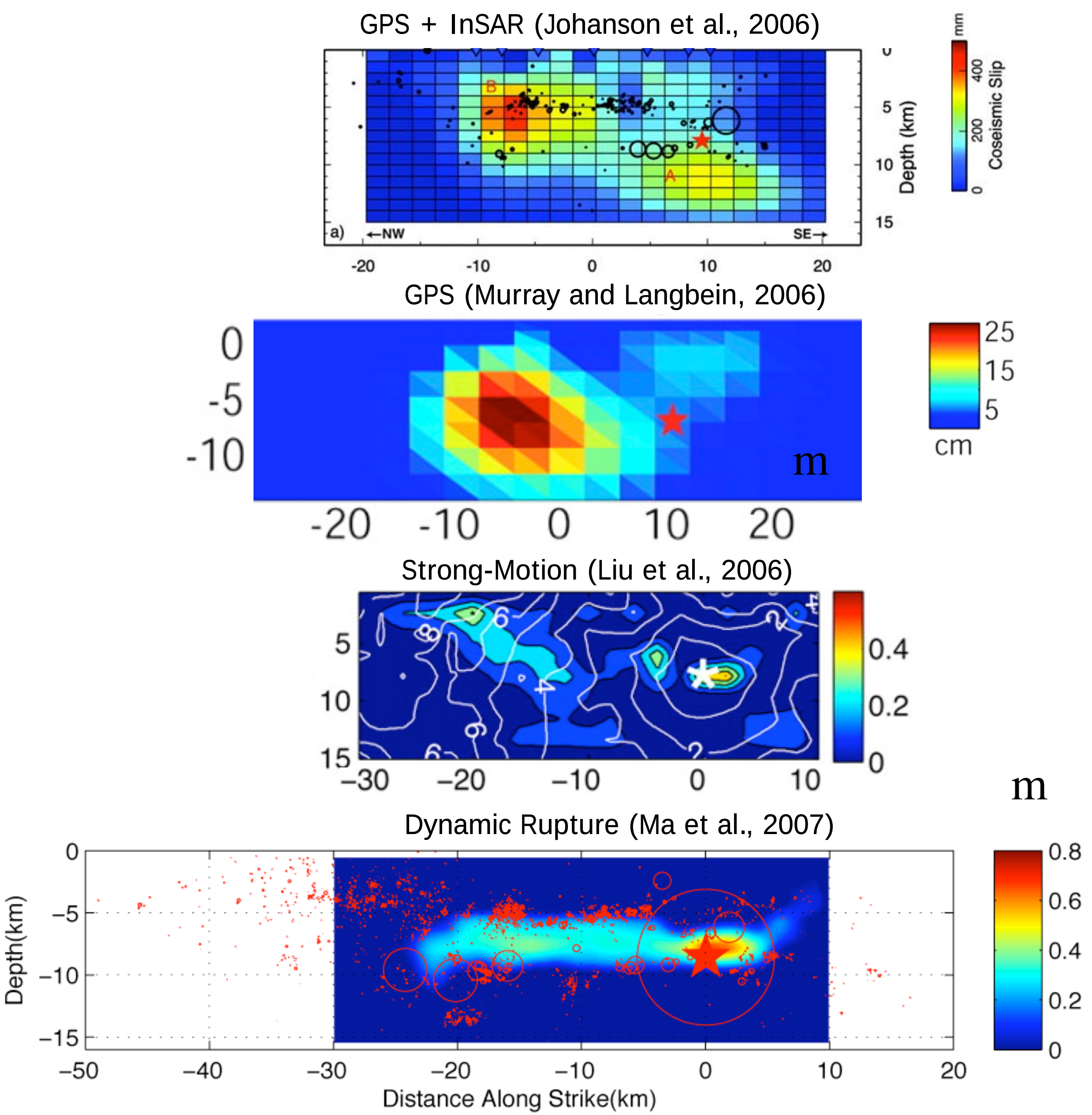
- Regularize by smoothing in space and/or time
- Interpolate model on coarse grid
- Low-pass data
- Limit rupture time to one or a few time steps
- Assume the form of the slip function
- Perform non-linear search for rupture front
- etc.

Comparison of Models for 2004 Parkfield Earthquake



A Path Forward with Minimal Assumptions

- Grid model (space and time) consistent with frequency content of the data.
- Use as short a seismogram as possible.
- Model displacement seismograms to capture near field.
- Apply a causality condition but no other constraint on rupture evolution.
- Regularize solution by smoothing.

This leads to very large systems of equations (~10^5 or more unknowns) for moderate-sized earthquakes.
Solution methods include LSQR, NNLS (if small enough) and SIRT (with or without non-negativity constraints).

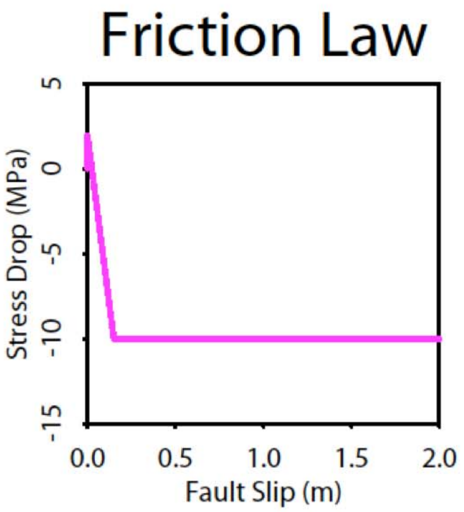
Assessment of the solution is an issue, as the problems is too large to use a.b.i.c., and LSQR underestimates the uncertainty. (Better methods are needed.)

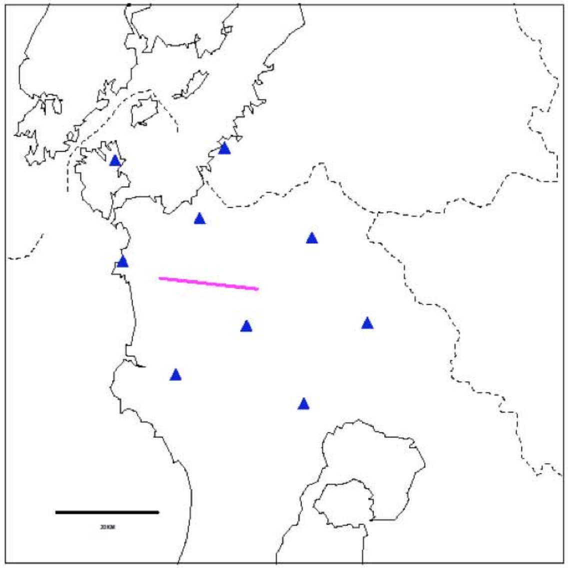
Comparison of Modeling for L'Aquila Earthquake

	Cirella, et al.	Ellsworth & Chiaraluce
Element Size	3.5 Km	0.2 – 0.4 km
Frequency Band	0.02 – 0.5 Hz	D.C. to 4 Hz
Velocity Structure for Green's Fn.	Plane Parallel Layers	Half Space
Source Time Function	Yofi Function (assumed)	Unconstrained
Rupture Time	Constrained (one time only)	Unconstrained
Data Duration	60 s	4 – 8 s

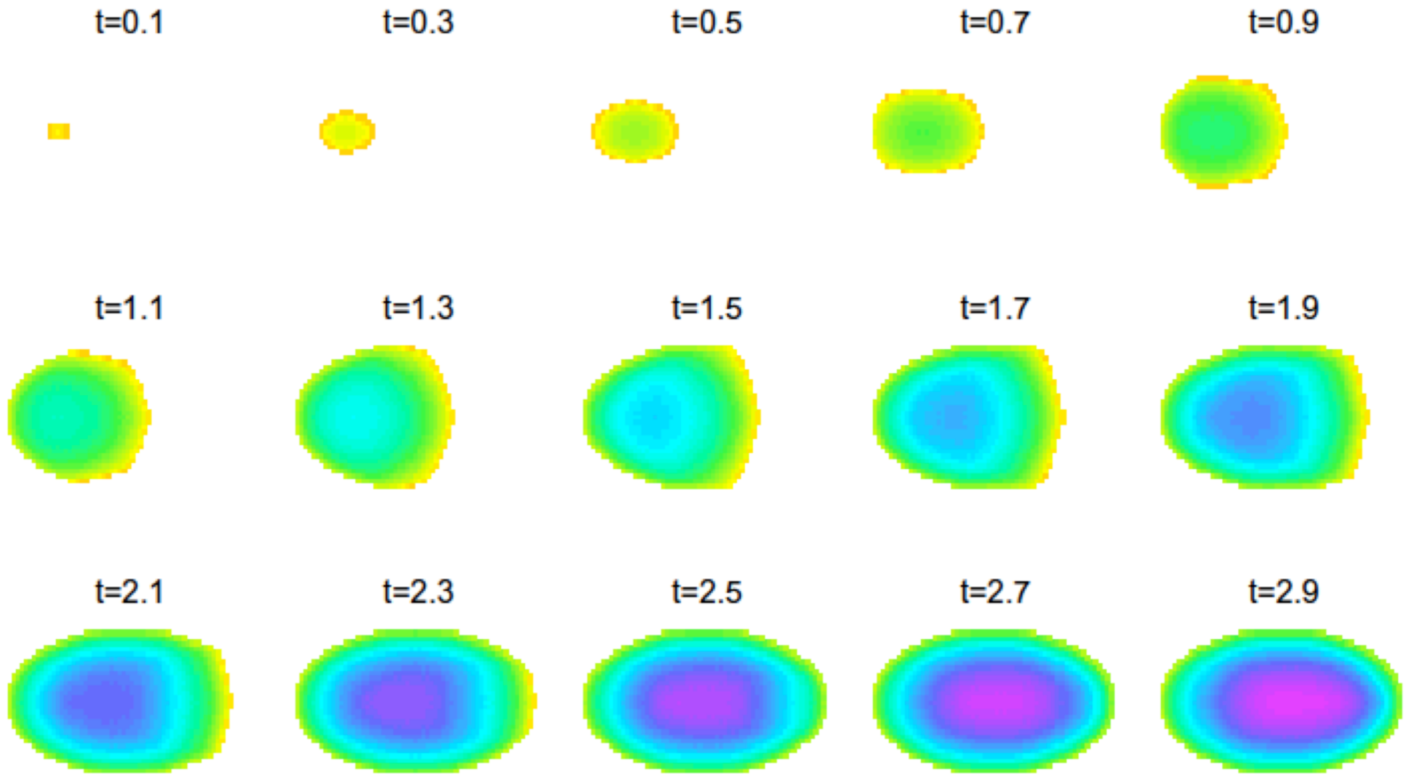
Synthetic Test Problem

- Displacement weakening friction with Dc fixed
- Forward problem solved using boundary element method
- Use realistic station geometry
- Invert displacement seismograms
- Compare regularization strategies



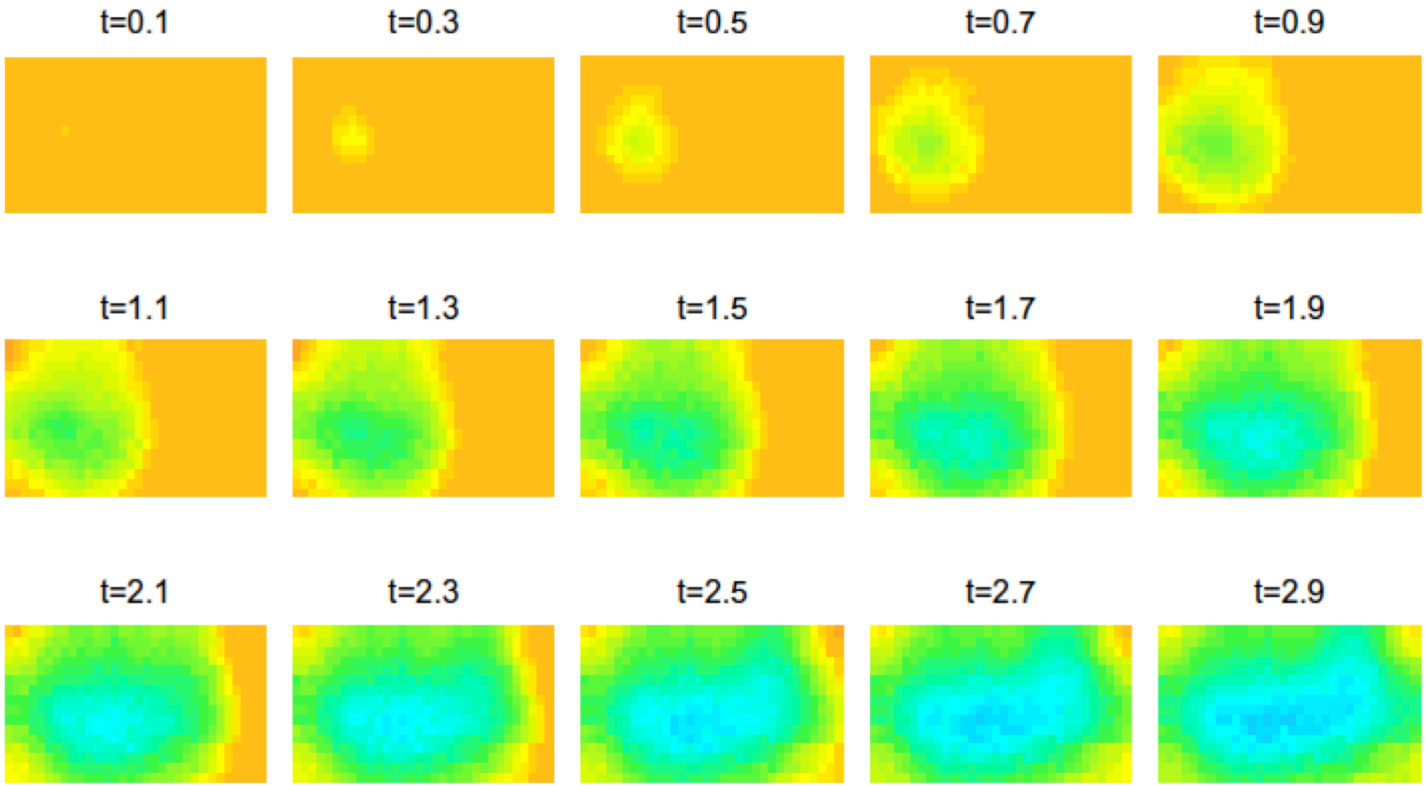


Dc = 0.05 m

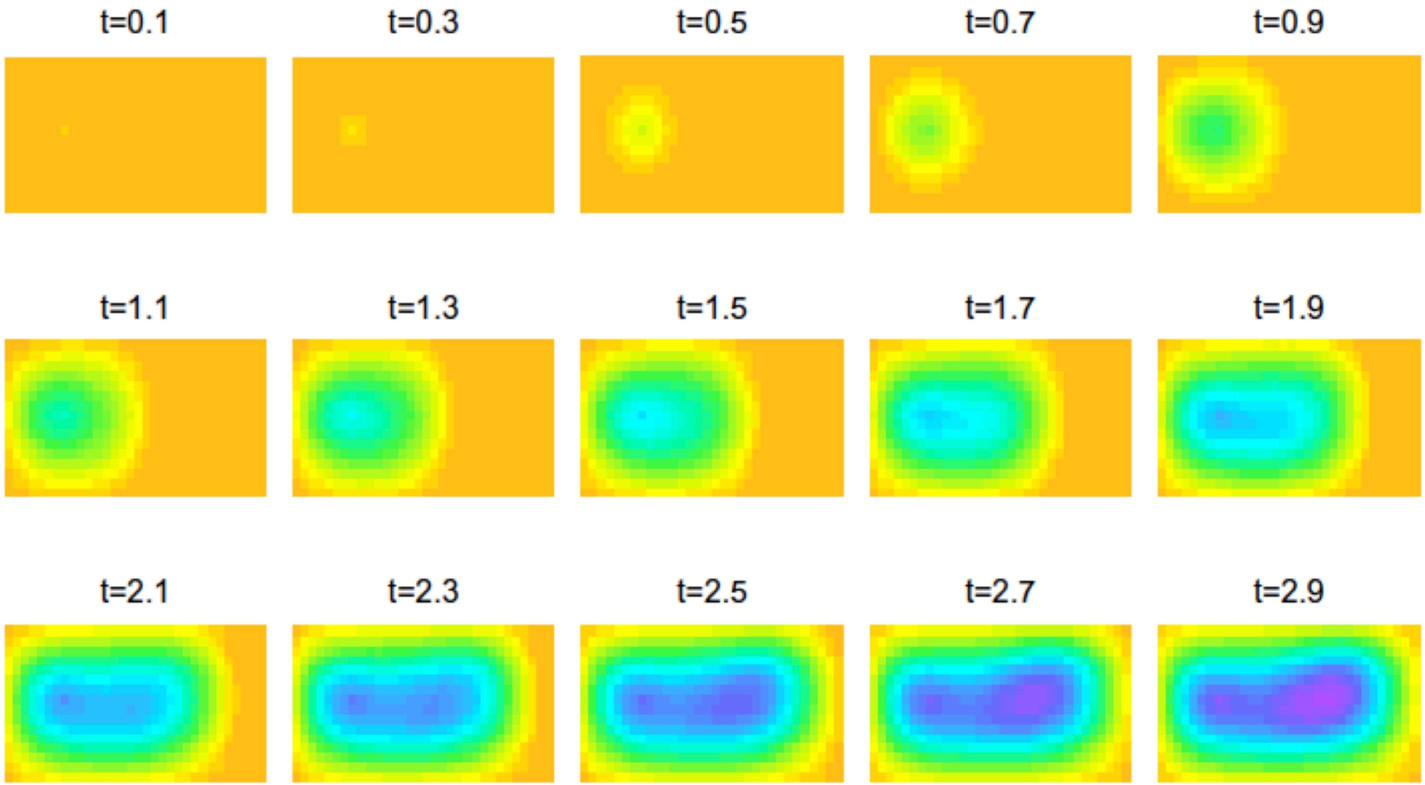


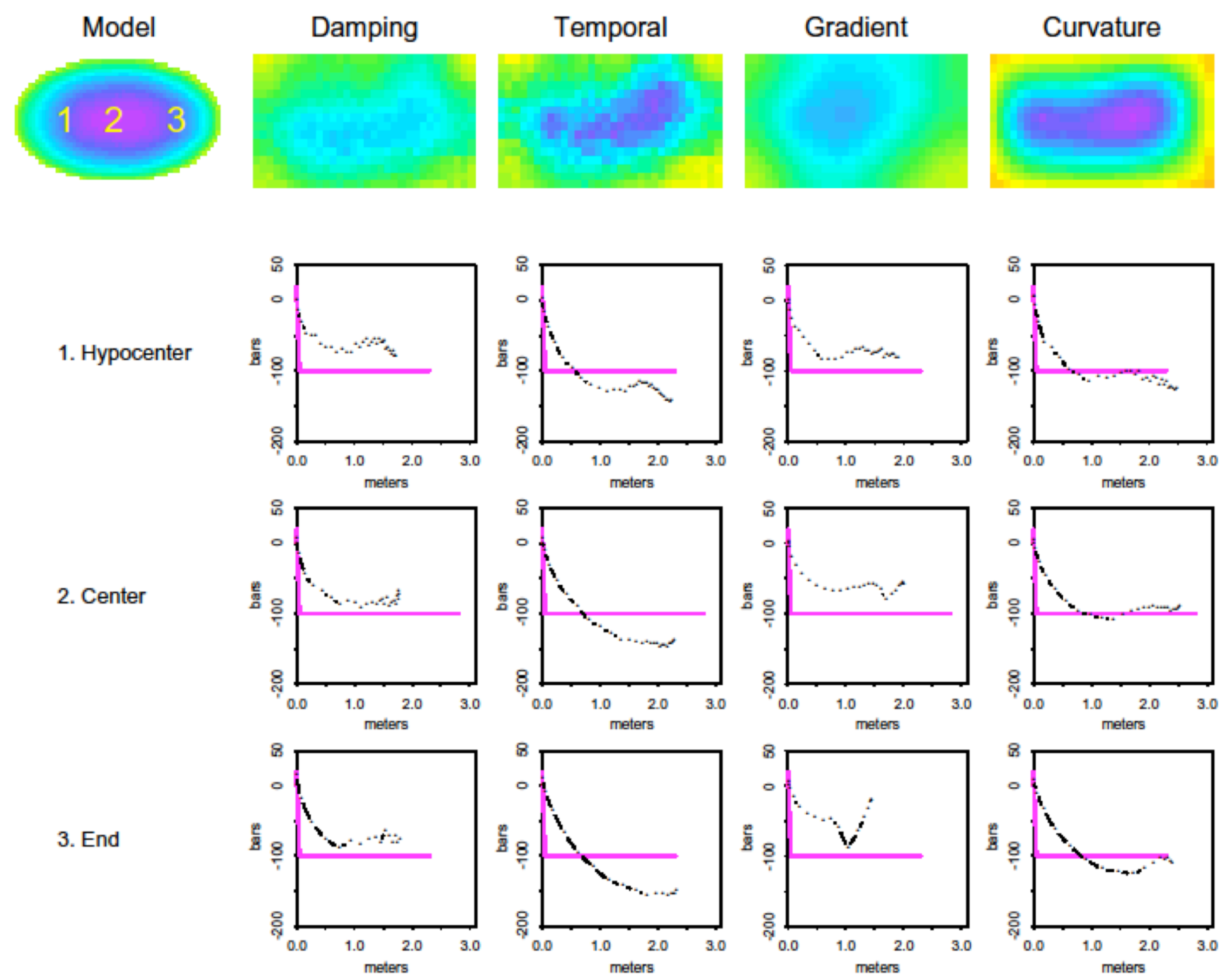
Damped Least-Squares Regularization

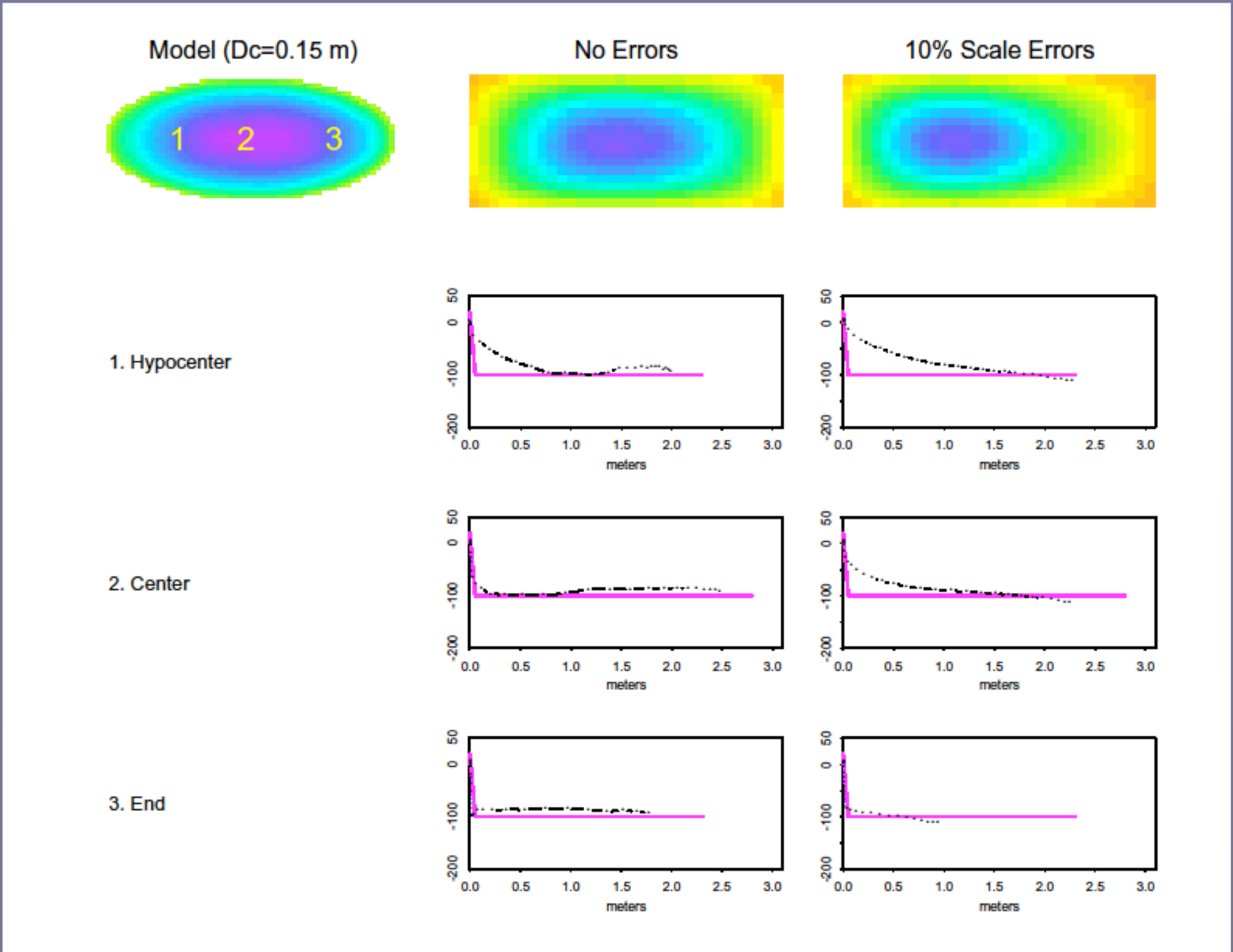
Dc = 0.05 m



Curvature (Laplacian) Regularization
Dc = 0.05 m




Model
Damping
Temporal
Gradient
Curvature
1 2 3
1. Hypocenter
2. Center
3. End
bars
meters
50
0
-100
-200
0.0
1.0
2.0
3.0



Model (Dc=0.15 m)
1
2
3
No Errors
10% Scale Errors
1. Hypocenter
2. Center
3. End
50
0
-100
-200
0.0
0.5
1.0
1.5
2.0
2.5
3.0
meters

Summary of Synthetic Tests

- Regularization using curvature (Laplacian) did the best job of recovering the total displacement in most tests.

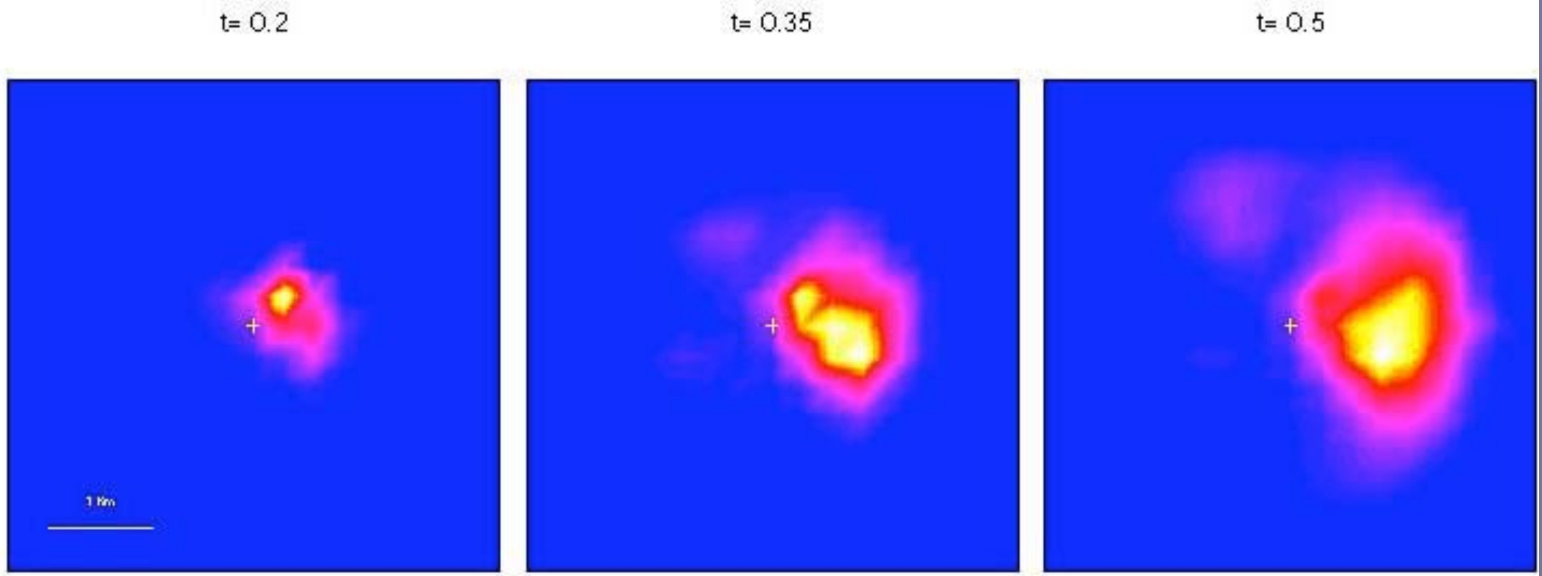
- Neither damping nor the spatial gradient regularization worked very well.

- Friction law could not be recovered reliably.
-
- All models seriously overestimate D_c.

Quick Look at Three Earthquakes:

- M 6.8 1995 Kobe (3 s of data for nucleation)
- M 6.3 2009 L'Aquila (8 s of data)
- M 6.0 1997 Northwest Kagoshima (5 s of data)

1995 Kobe Earthquake Initially Grows Like a Unilaterally Expanding Crack

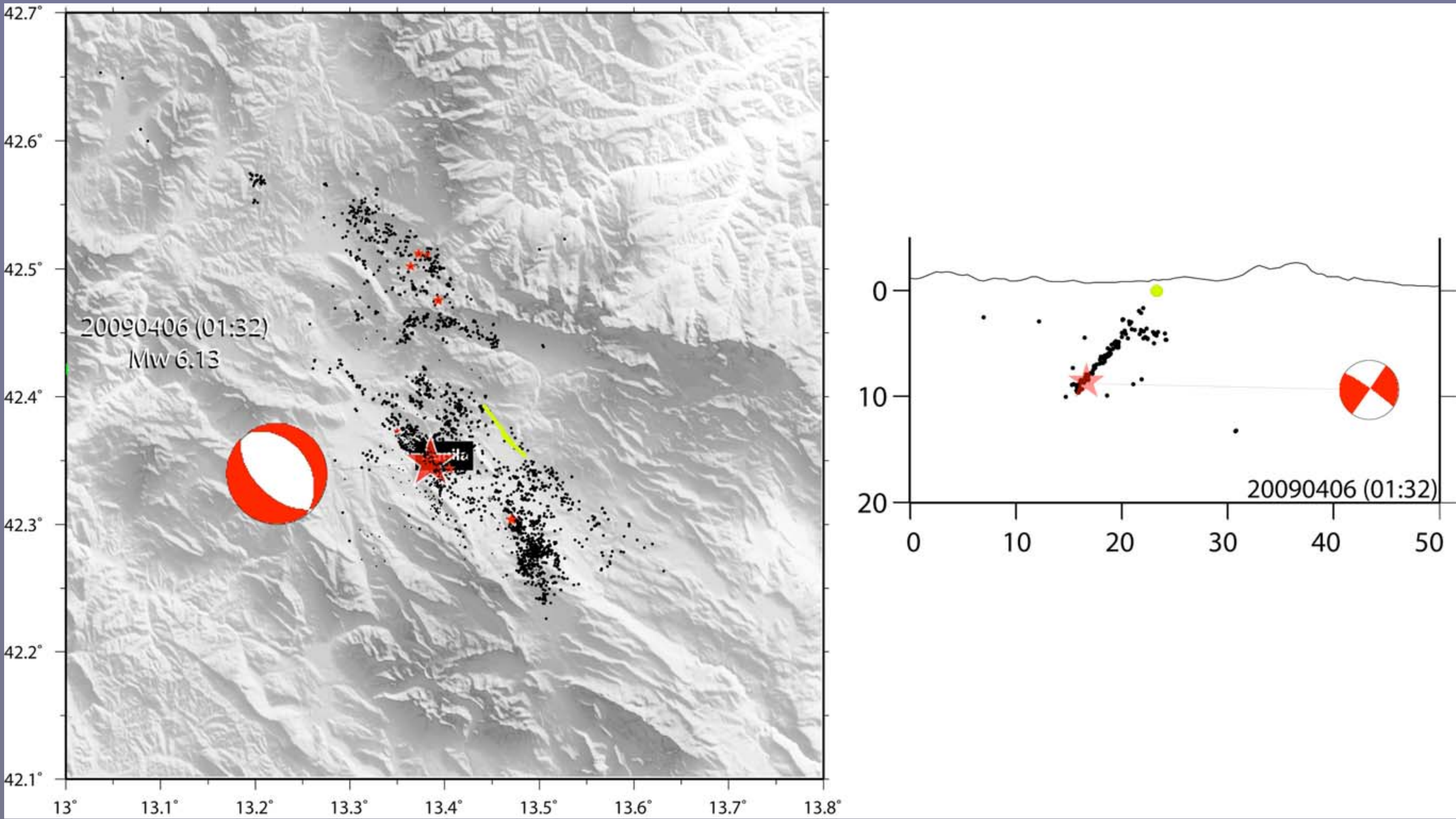


Cumulative displacement

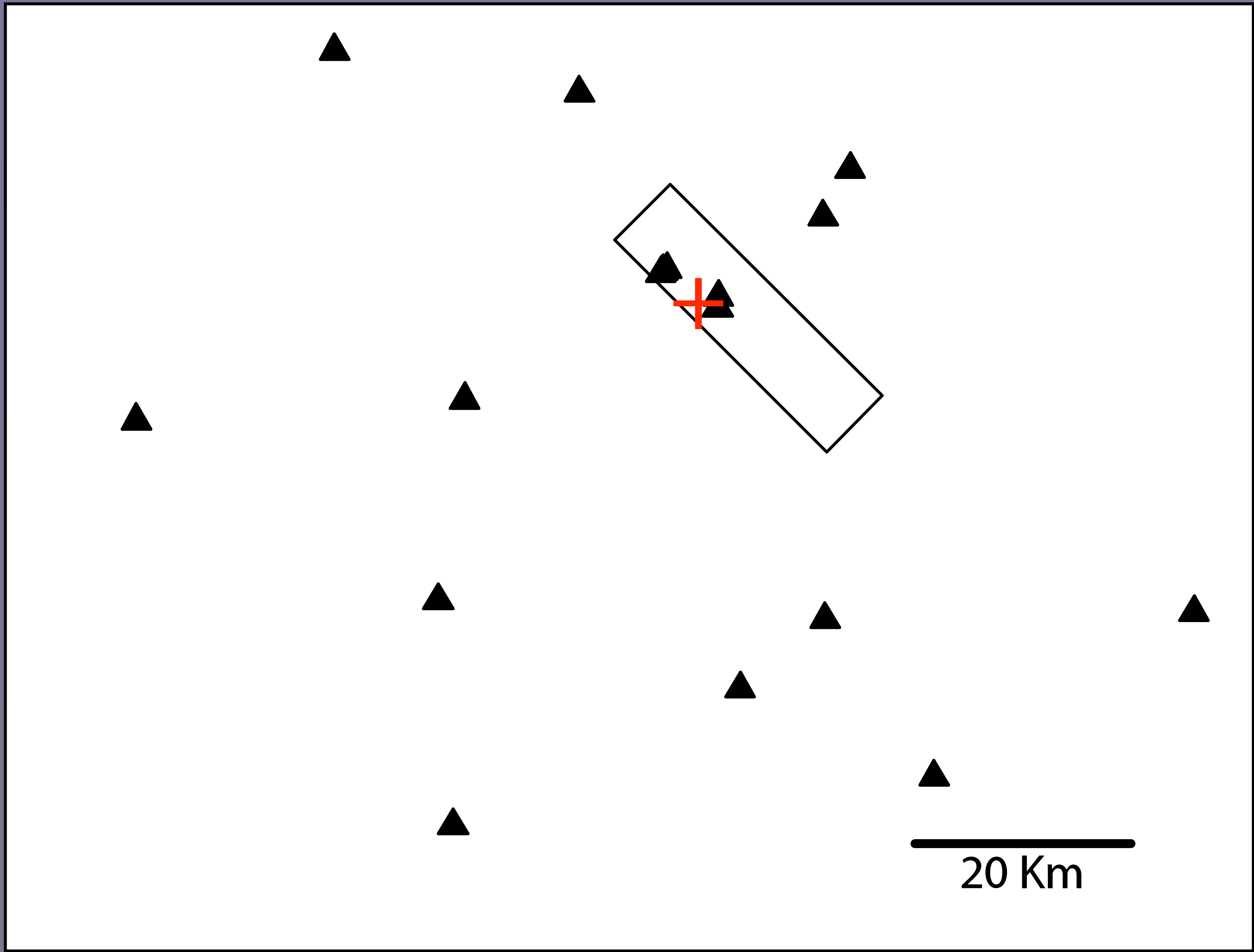
L’Aquila Italy Earthquake

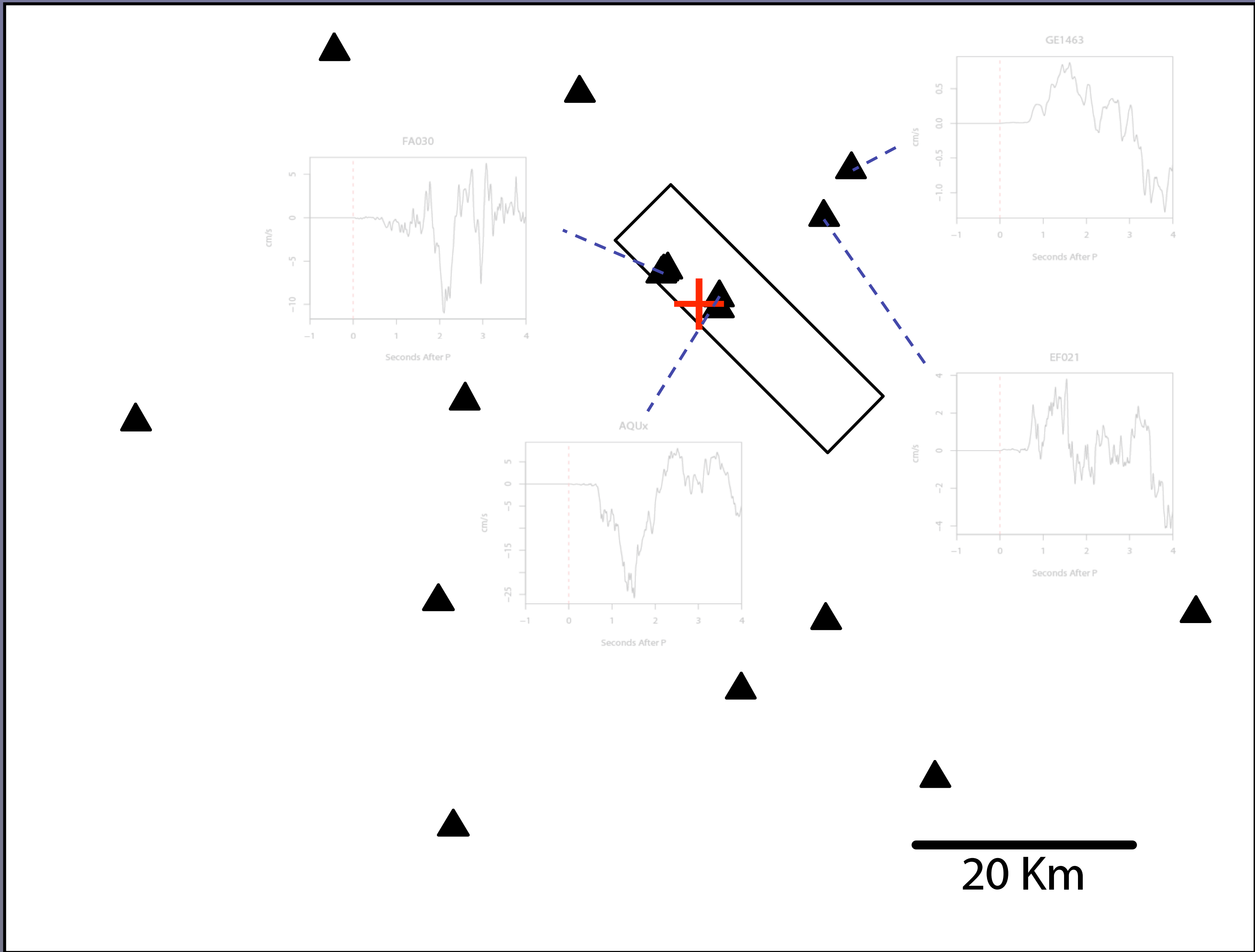
April 6, 2009

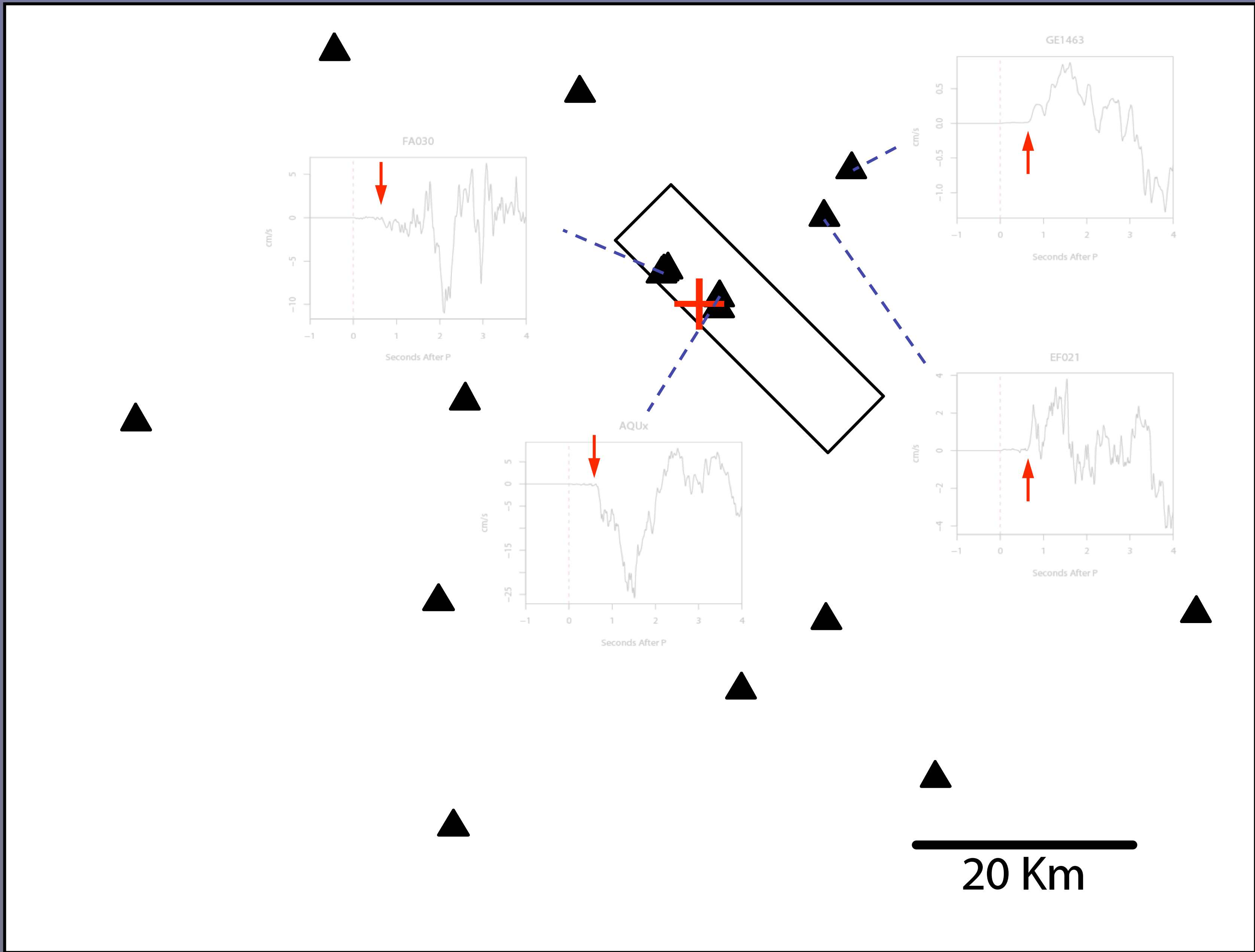
Mw 6.3



Strong Motion Stations

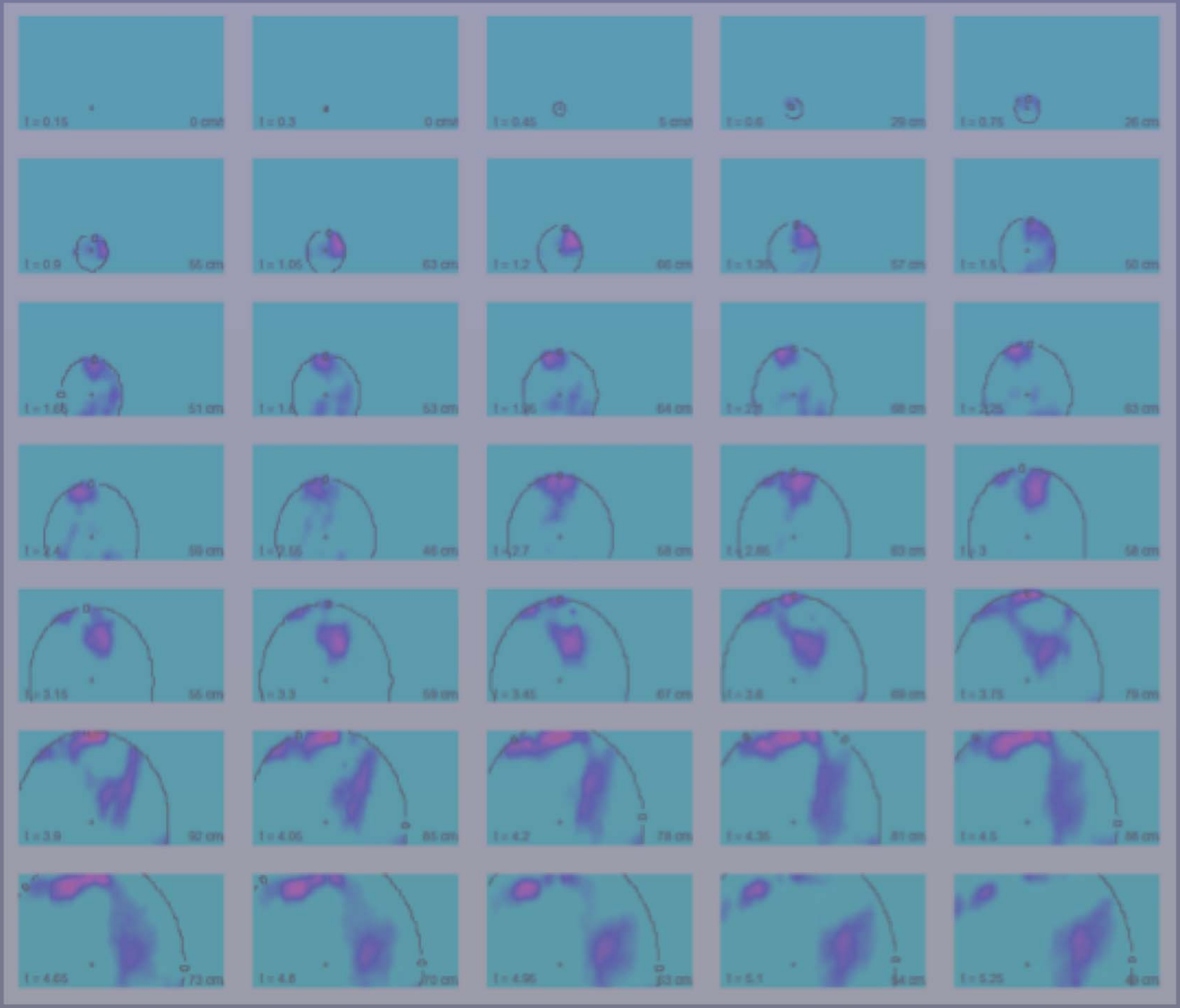



GE1463
FA030
EF021
AQUx
Seconds After P
20 Km

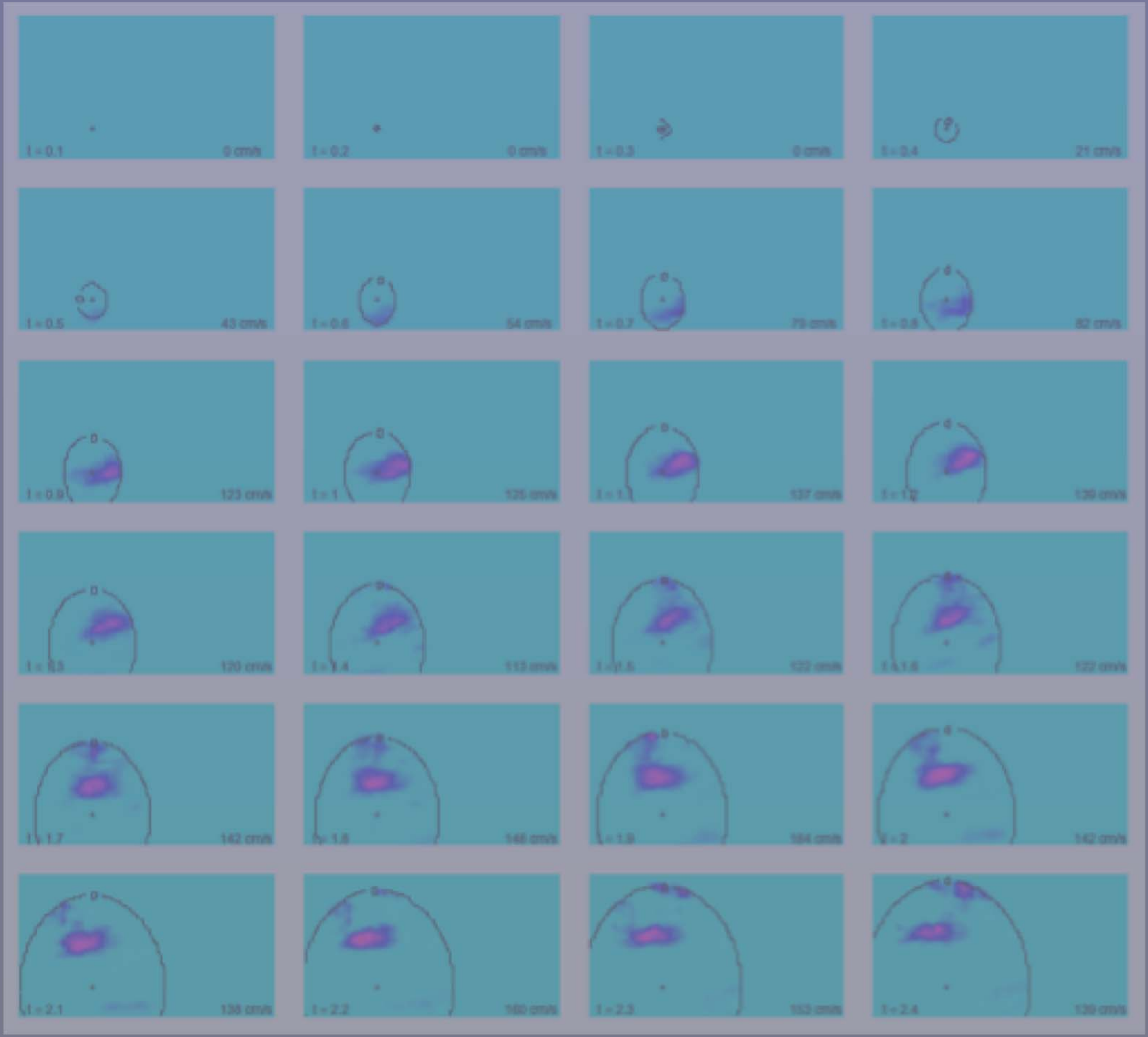


FA030
cm/s
Seconds After P
GE1463
cm/s
Seconds After P
AQUx
cm/s
Seconds After P
EF021
cm/s
Seconds After P
20 Km

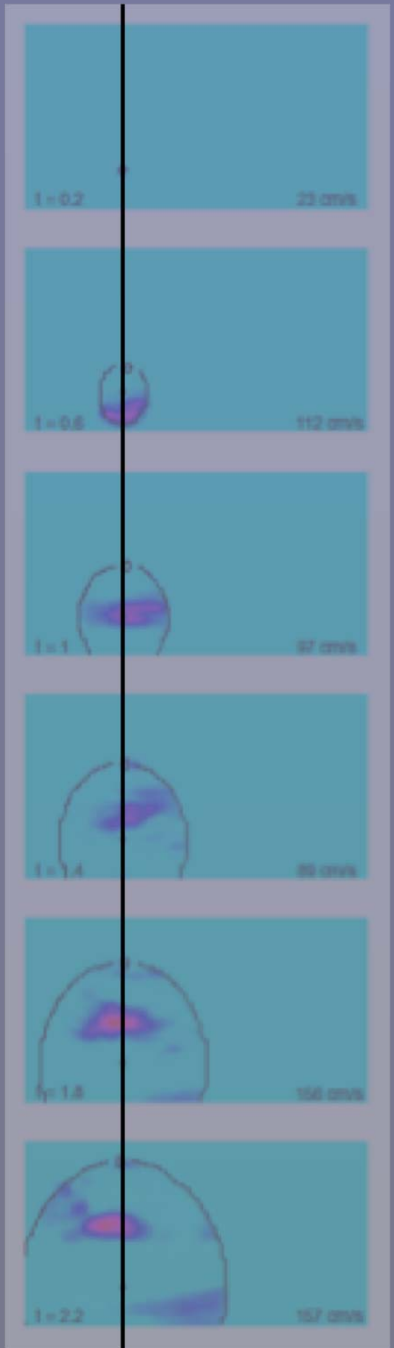
Snapshots of Slip Velocity on the Fault Plane in this study

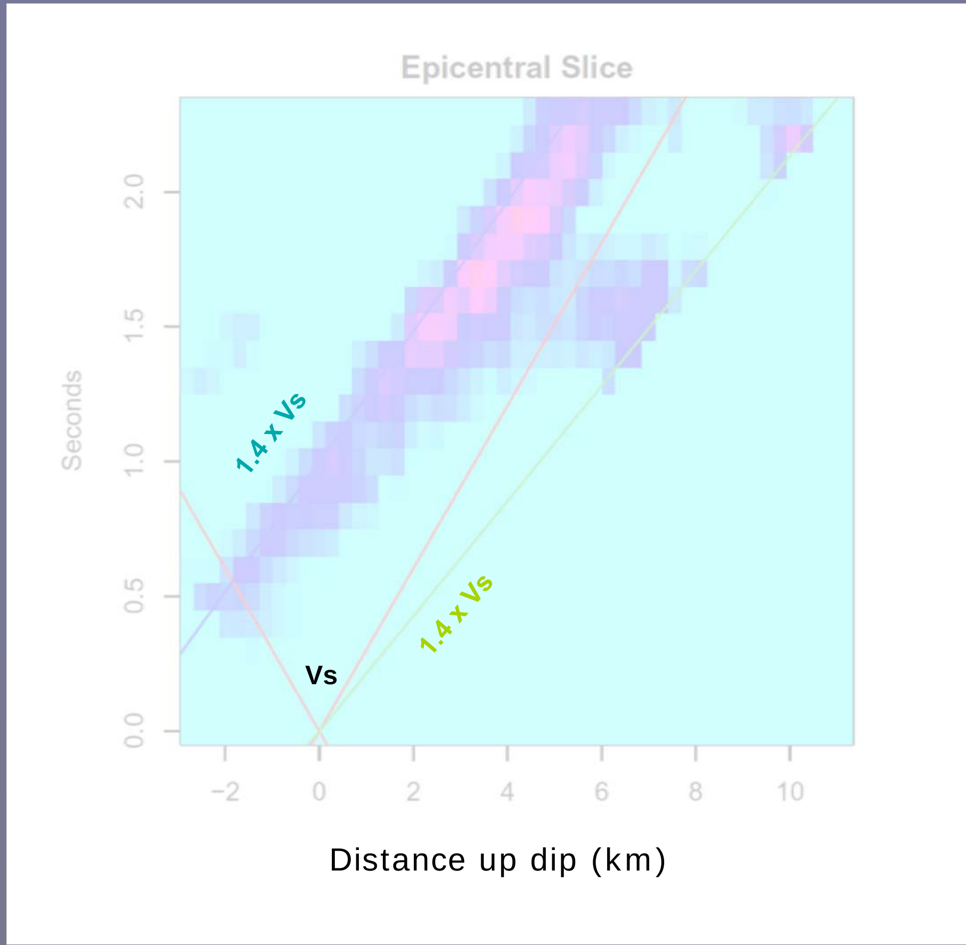


Details of Nucleation and Breakaway

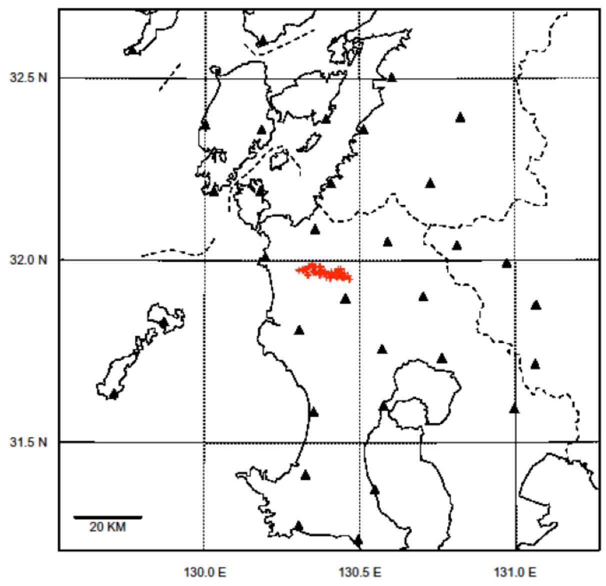


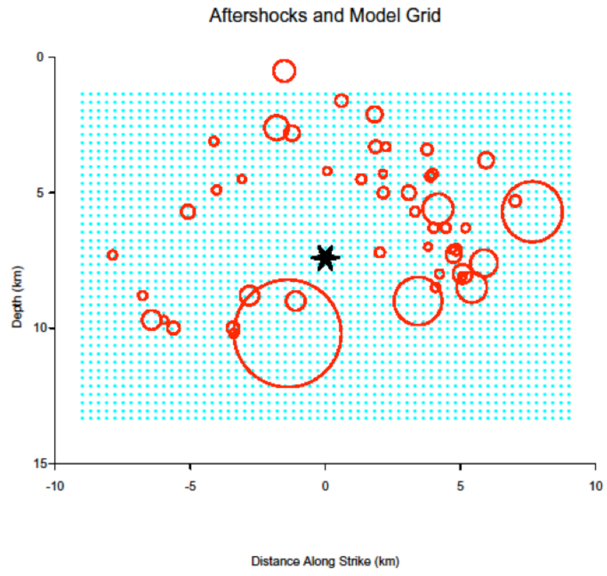
Supershear rupture initiates at breakaway (+0.6 s) and propagates up dip as a slip pulse

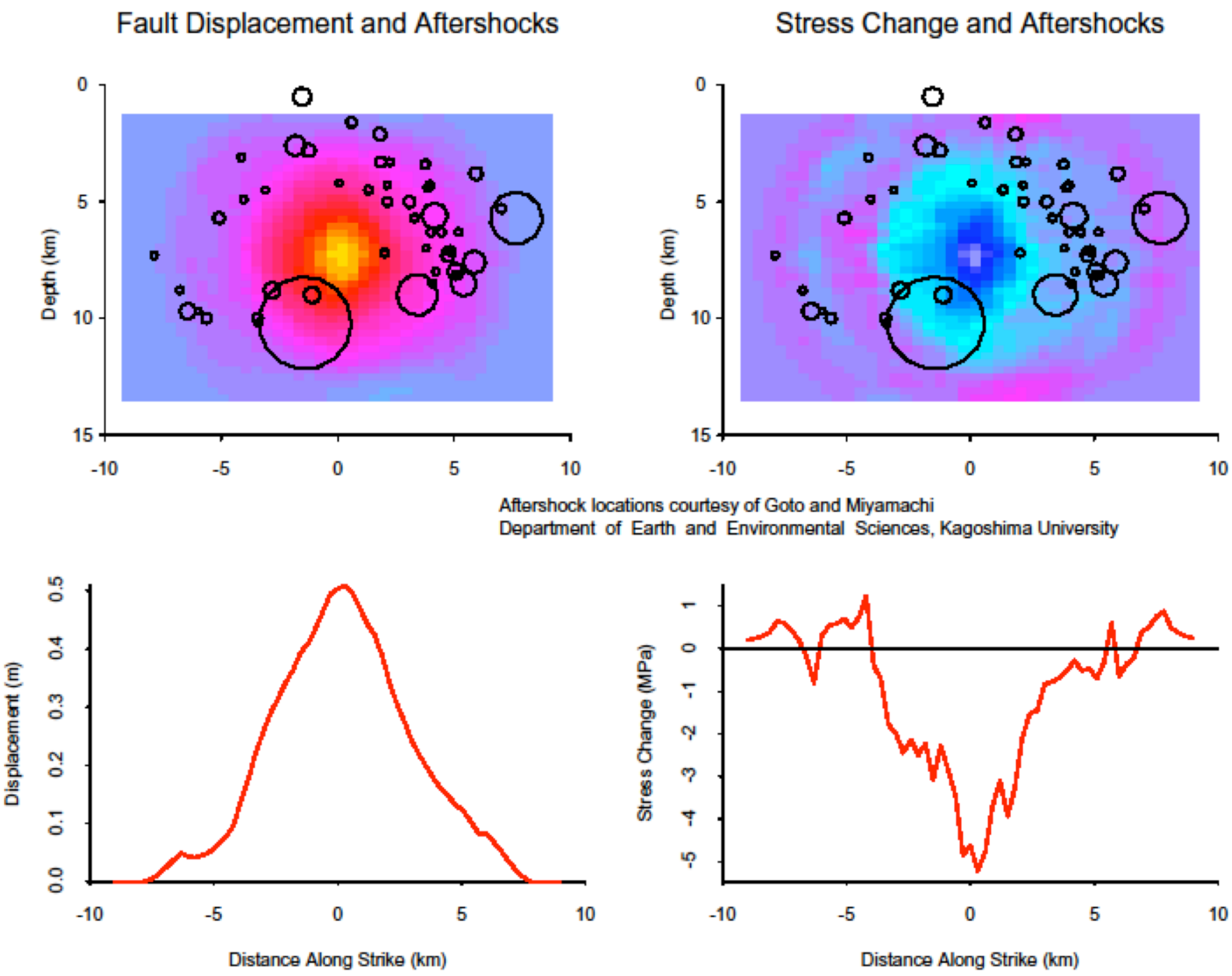




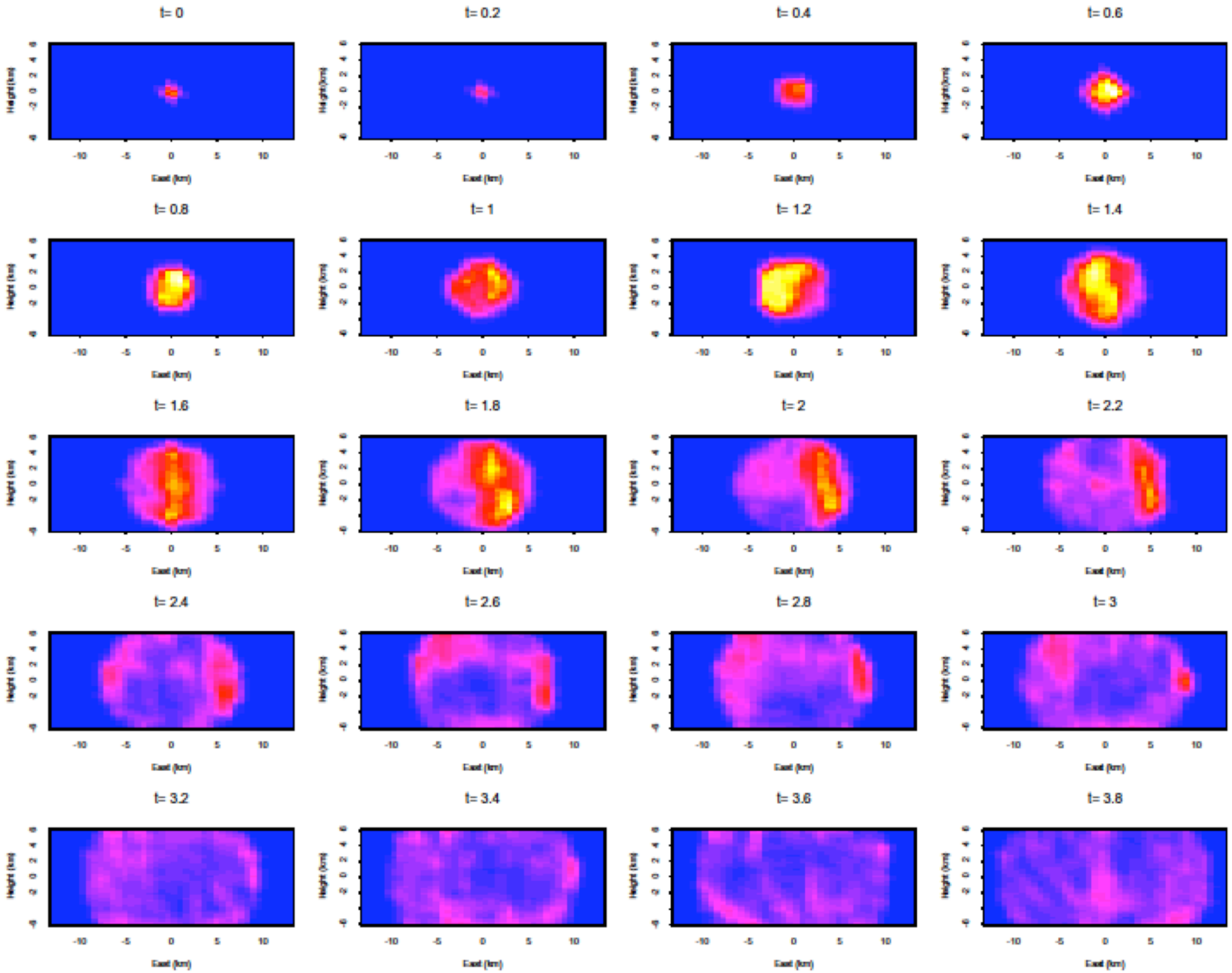
1997 M 6.0 Northwest Kagoshima, Japan

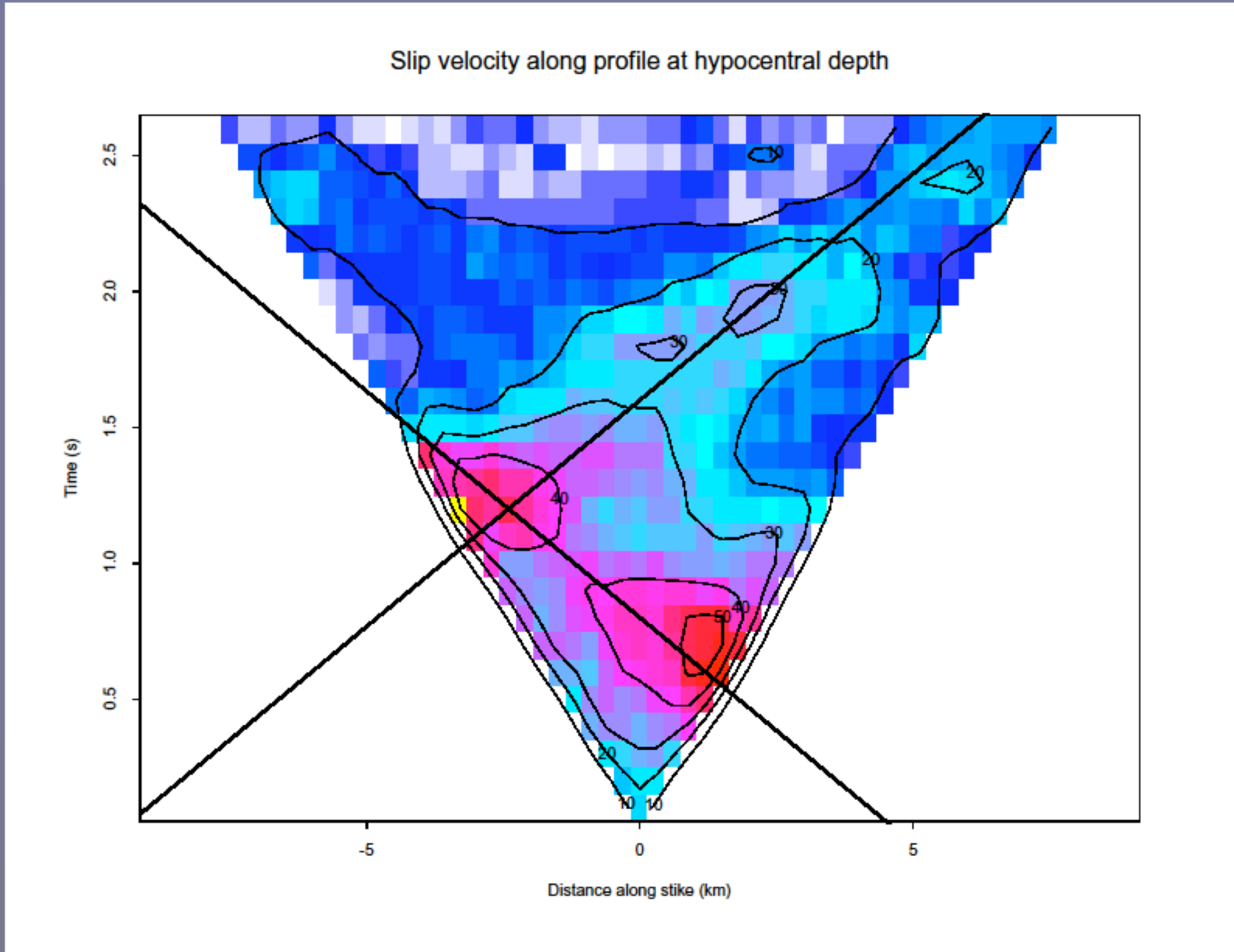






Aftershock locations courtesy of Goto and Miyamachi
Department of Earth and Environmental Sciences, Kagoshima University


t= 0
t= 0.2
t= 0.4
t= 0.6
t= 0.8
t= 1
t= 1.2
t= 1.4
t= 1.6
t= 1.8
t= 2
t= 2.2
t= 2.4
t= 2.6
t= 2.8
t= 3
t= 3.2
t= 3.4
t= 3.6
t= 3.8
East (km)
Height (km)


Slip velocity along profile at hypocentral depth
Time (s)
2.5
2.0
1.5
1.0
0.5
Distance along stike (km)
-5
0
5
10
20
30
40
50

SUMMARY

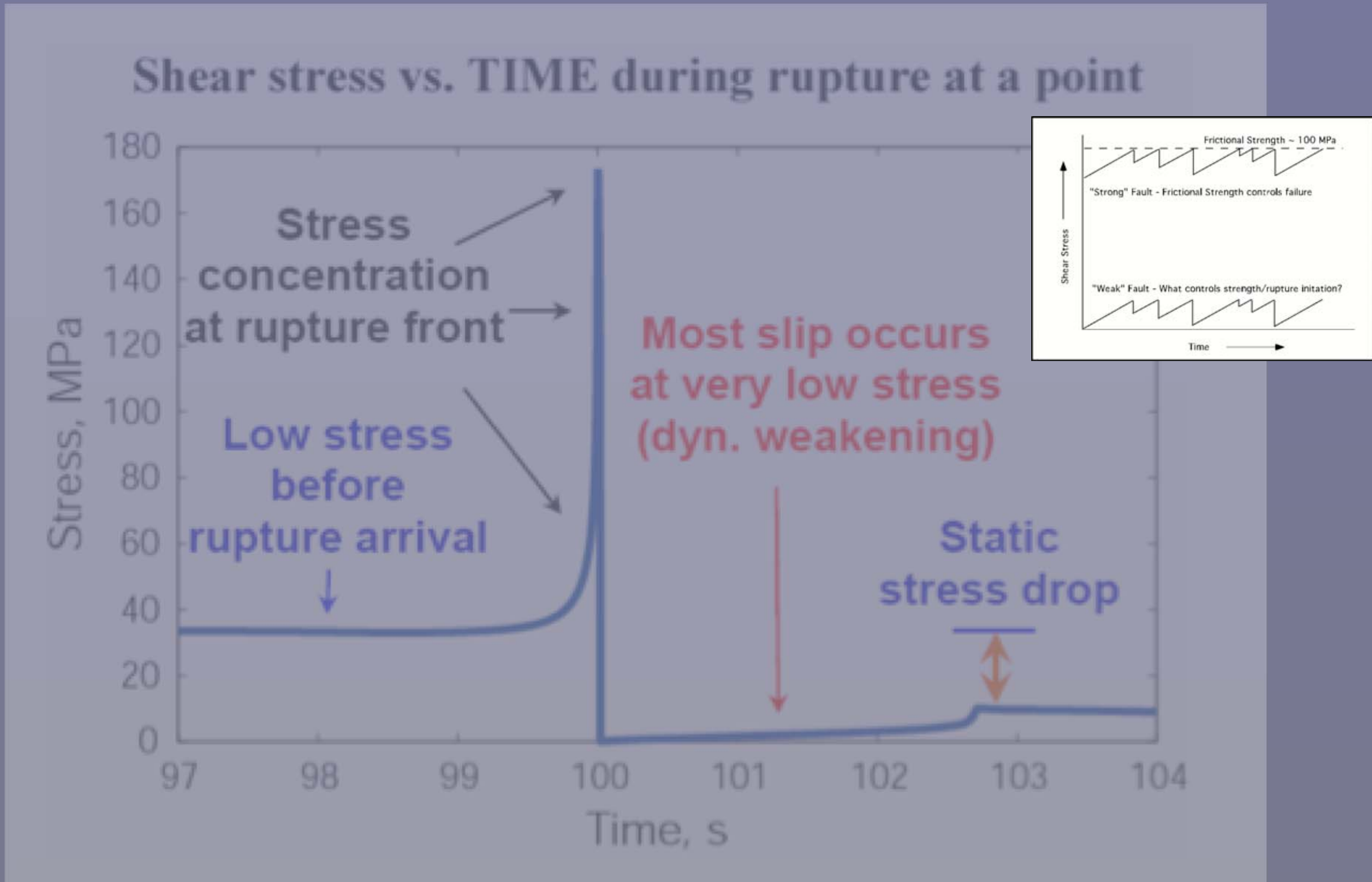
Kinematics of the rupture can be recovered by solving the linear inverse problem with spatial (Laplacian) smoothing.

No pre-conditioning of the rupture aside from causality is needed.

Near-field displacement seismograms are particularly valuable for this purpose.

Very short data records can be used to focus on details of rupture initiation.

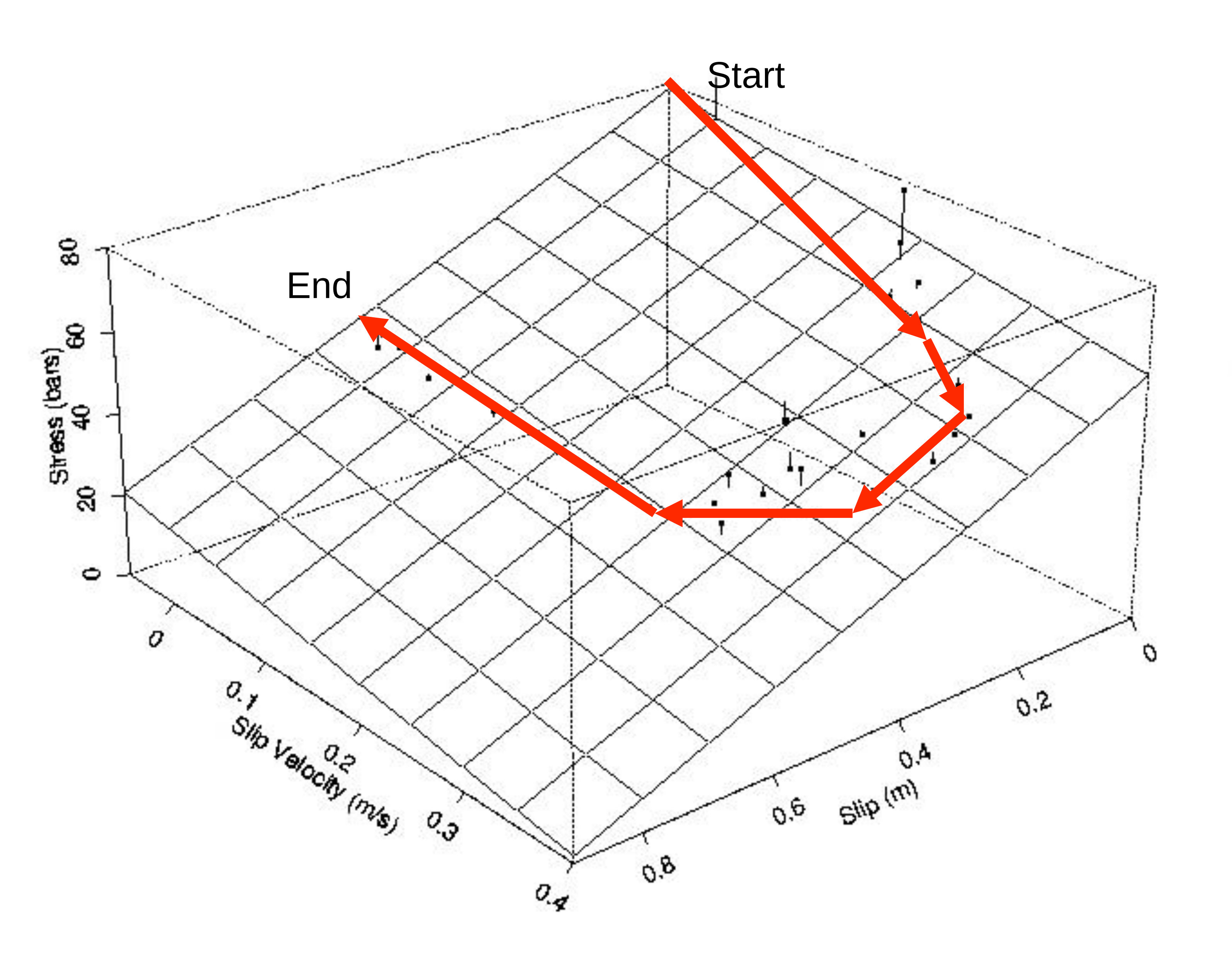
L'Aquila and N.W. Kagoshima display Mode II supershear slip pulses propagating through and/or beyond the already ruptured fault.



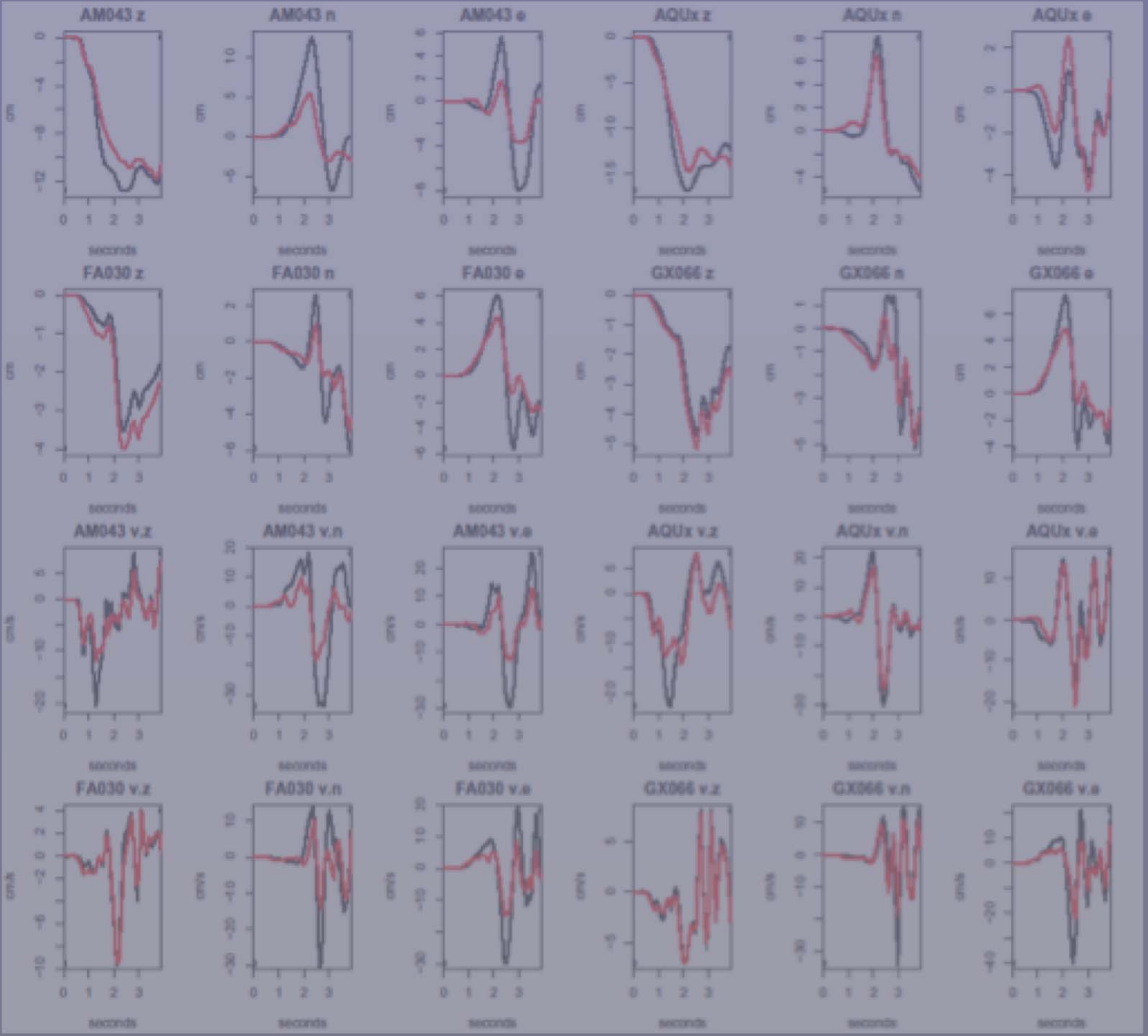
(Lapusta and Rice, JGR, in prep.)

Nadia Lapusta

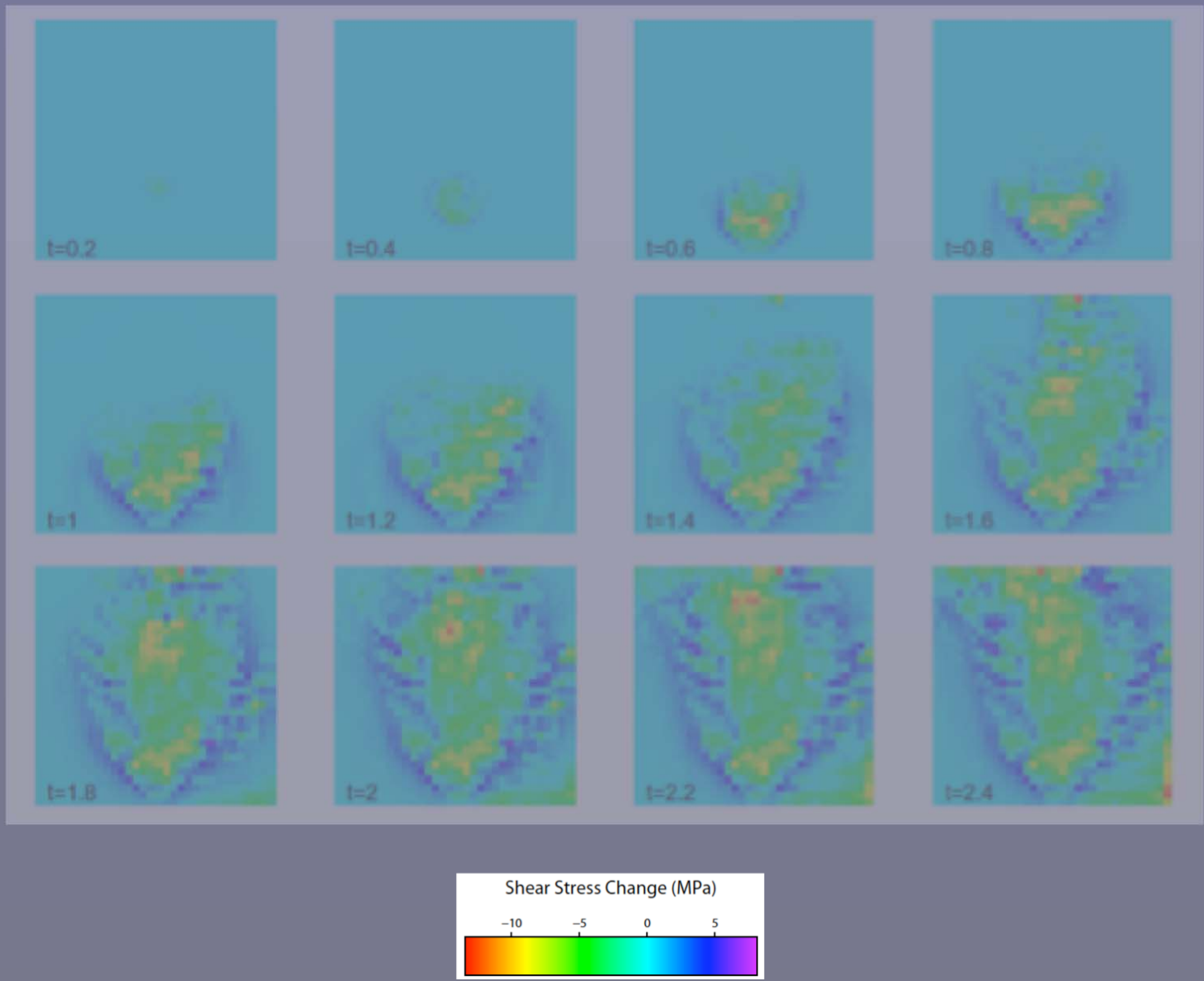
Earthquake Slip Trajectory in “Phase Space”



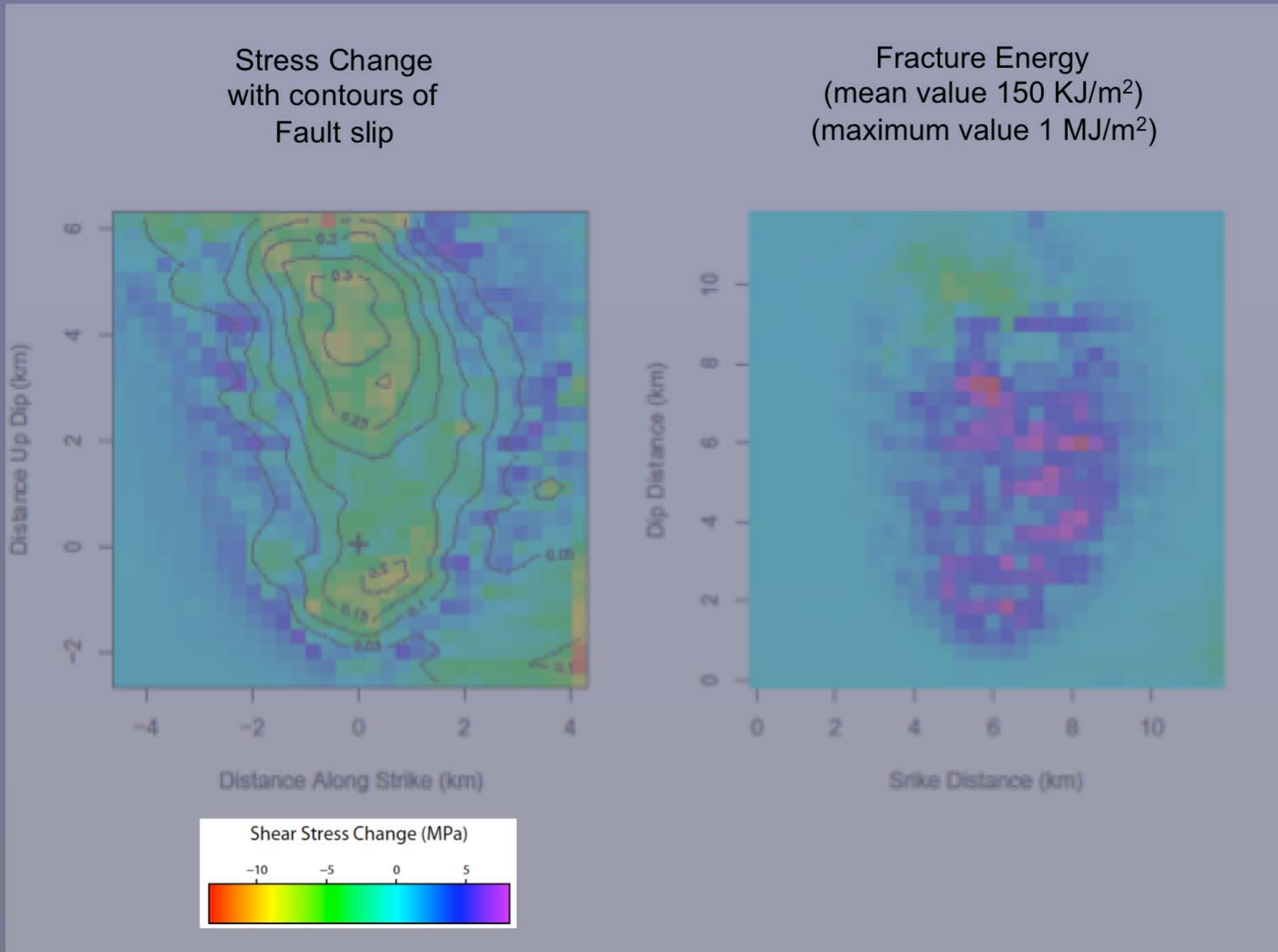
Data and Fit


AM043 z
AM043 n
AM043 e
AQUx z
AQUx n
AQUx e
FA030 z
FA030 n
FA030 e
GX066 z
GX066 n
GX066 e
AM043 v.z
AM043 v.n
AM043 v.e
AQUx v.z
AQUx v.n
AQUx v.e
FA030 v.z
FA030 v.n
FA030 v.e
GX066 v.z
GX066 v.n
GX066 v.e
seconds

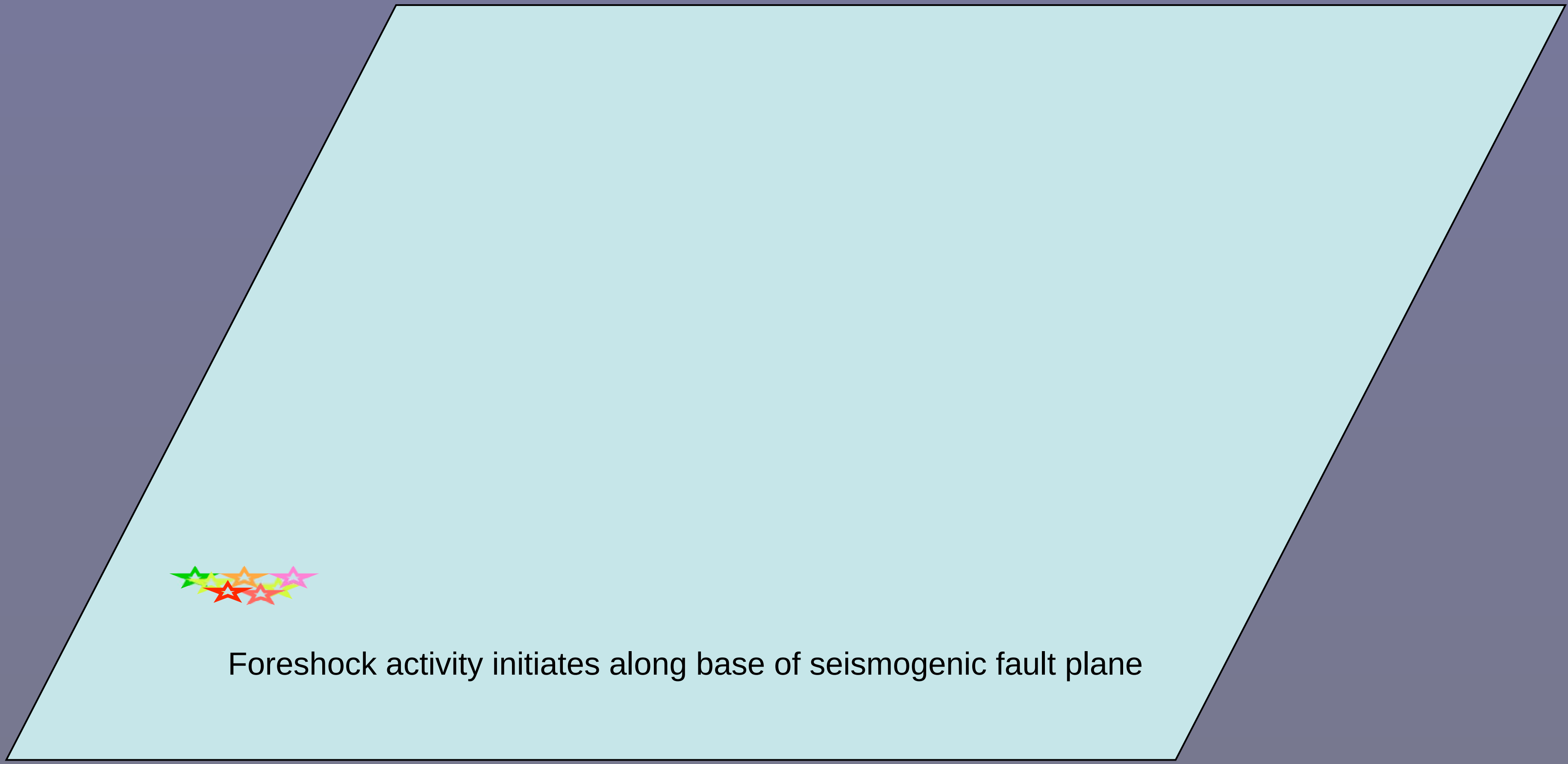
Shear Stress Change on the Fault Plane
During Nucleation and Breakaway



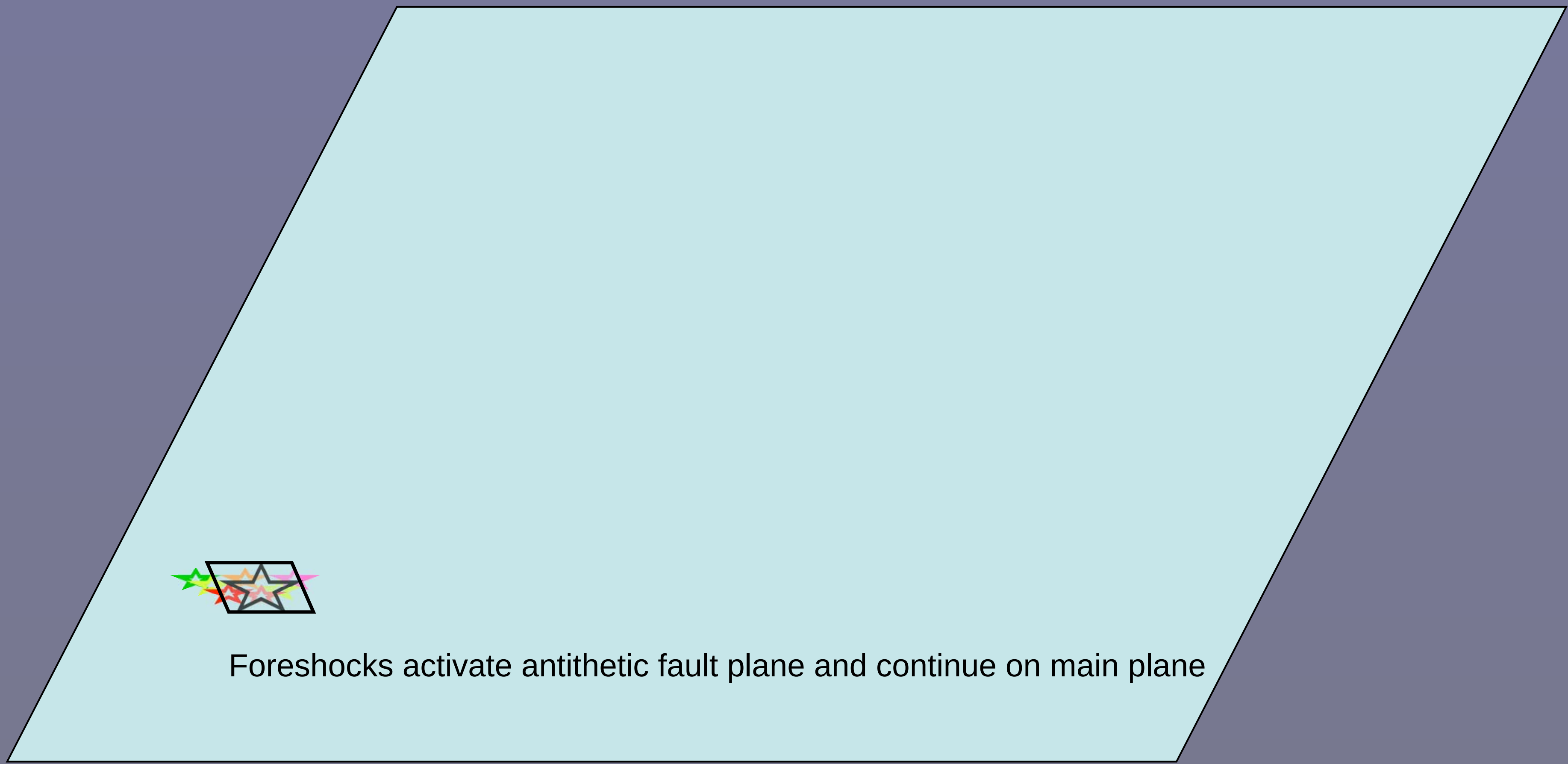
State of the Fault at +2.4 s into the Earthquake



January – March 2009

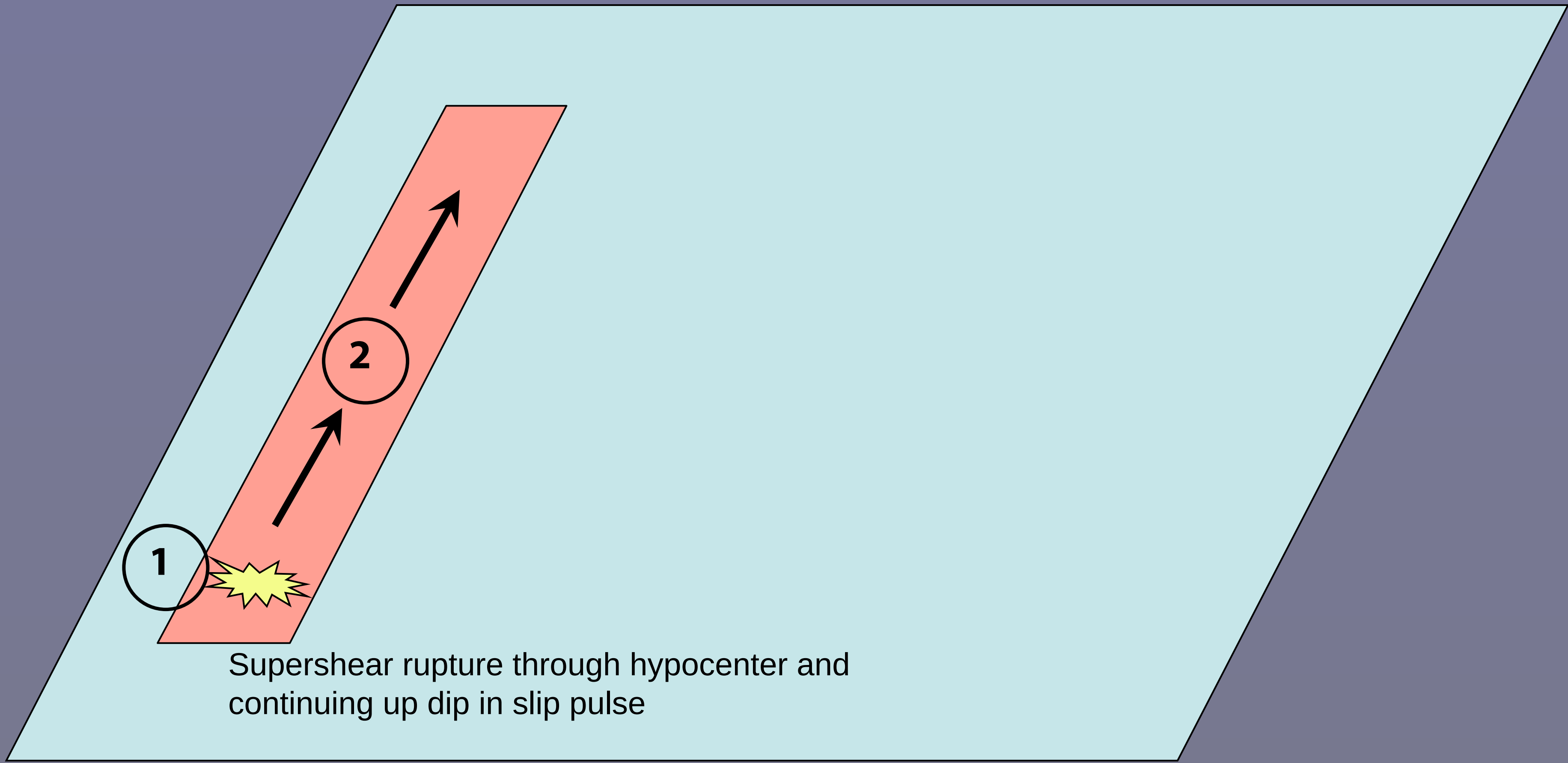


March 30 – April 6 2009

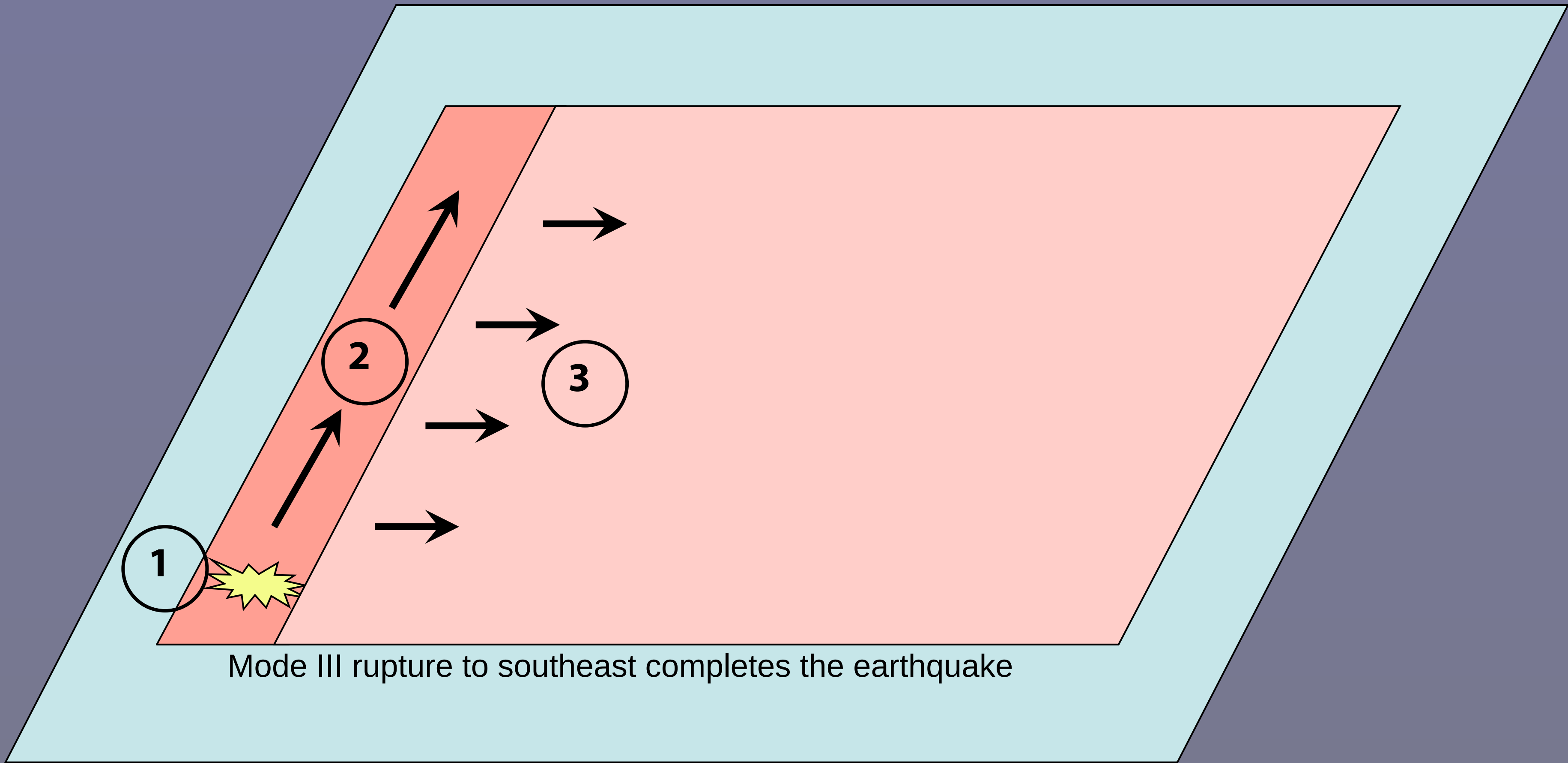


April 6, 2009 0.0 – 0.6 seconds

April 6, 2009 0.6 – 2.4 seconds



April 6, 2009 2.4 – 8 s



Mode III rupture to southeast completes the earthquake